California
Subject
Examinations for
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# TEST GUIDE

## MATHEMATICS
## SUBTEST I

## Sample Questions and Responses and Scoring Information



CS-TG-QR211-01

# Sample Test Questions for CSET: Mathematics Subtest I

Below is a set of multiple-choice questions and constructed-response questions that are similar to the questions you will see on Subtest I of CSET: Mathematics. Please note that, as on the actual test form, approximately one third of the multiple-choice questions in this test guide are more complex questions that require 2–3 minutes each to complete. You are encouraged to respond to the questions without looking at the responses provided in the next section. Record your responses on a sheet of paper and compare them with the provided responses.

Note: The use of calculators is not allowed for CSET: Mathematics Subtest I.

Note: In CSET: Mathematics subtests, log $x$ represents the base-10 logarithm of $x$.

1. **Use the table below to answer the question that follows.**

| @ | ***a*** | ***b*** | ***c*** | ***d*** |
|---|---|---|---|---|
| ***a*** | *c* | *a* | *b* | *d* |
| ***b*** | *a* | *b* | *c* | *d* |
| ***c*** | *b* | *c* | *a* | *d* |
| ***d*** | *d* | *d* | *d* | *d* |

The set $M = \{a, b, c, d\}$ under operation @ is defined by the table shown above. Which of the following is the inverse element for $a$ under operation @?

A. $a$

B. $b$

C. $c$

D. $d$

2. Which of the following expressions is equivalent to
$\dfrac{25 \cdot (2)^{10b - 8}}{10 \cdot (2)^{5b + 2}}$?

A. $5 \cdot (2)^{2b - 5}$

B. $5 \cdot (2)^{5b - 11}$

C. $15 \cdot (2)^{2b - 4}$

D. $15 \cdot (2)^{5b - 10}$

3. Three numbers, $x$, $y$, and $z$, have a sum of 871. The ratio $x$:$y$ is 4:5 and the ratio $y$:$z$ is 3:8. Which of the following is the value of $y$?

A. 134

B. 156

C. 195

D. 201

4. Which of the following expressions is equivalent to $\dfrac{5 - i}{1 + 7i}$?

A. $5 - \frac{1}{7} i$

B. $-2 - \frac{17}{3} i$

C. $-\frac{1}{4} + \frac{3}{4} i$

D. $-\frac{1}{25} - \frac{18}{25} i$

5. In order to identify all the prime numbers less than 200, a person writes each number from 1 to 200, and eliminates all the multiples of 2, then all the multiples of 3. To complete this task, the person will have to eliminate the multiples of which additional numbers?

   A. 5, 7, 9, 11

   B. 7, 9, 11, 13

   C. 5, 7, 11, 13

   D. 7, 11, 13, 17

6. If $x$, $y$, and $z$ are nonnegative integers, what is the total number of factors of $2^x 3^y 5^z$?

   A. $(2 + 3 + 5)(x + y + z)$

   B. $xyz$

   C. $(x + 1)(y + 1)(z + 1)$

   D. $x^2y^3z^5$

7. **Use the theorem about integers below to answer the question that follows.**

   | |
   |---|
   | Given that $a > b$, $a = nb + r$, and that $d$ divides $a$ and $b$, then $d$ divides $r$. |

   Which of the following describes why this statement is true?

   A. If $d$ divides $a$ and $b$, then $d$ divides $a – b$.

   B. If $d$ divides $a$ and $b$, then $d$ divides $n$.

   C. If $d$ divides $a$ and $b$, then $d$ divides $ab$.

   D. If $d$ divides $a$ and $b$, then $d$ is the greatest common divisor of $a$ and $b$.

8. If $a$, $b$, $n$, and $r$ are positive integers such that $a > b$, and $a = nb + r$, then the greatest common divisor of $a$ and $b$ (gcd($a$, $b$)) is also equal to which of following?

   A. gcd($a$, $n$)

   B. gcd($b$, $r$)

   C. gcd($n$, $r$)

   D. gcd($n$, $b$)

9. Which of the following statements refutes the claim that $GL_R(3)$, the set of 3 × 3 invertible matrices over the real numbers, is a field?

   A. There exist elements $A$ and $B$ of $GL_R(3)$ such that $AB \neq BA$.

   B. There exist elements $A$ and $B$ of $GL_R(3)$ such that $\det(AB) = \det(A)\det(B)$.

   C. If $A$ is an element of $GL_R(3)$, then there exists a matrix $A^{-1}$ such that $A^{-1}A = I$.

   D. If $A$ is an element of $GL_R(3)$, then there exists a matrix $A$ such that $\det(A) \neq 1$.

10. It is given that the set of complex numbers $\mathbb{C}$ is a commutative ring with a multiplicative identity equal to 1. Let $z_1 = a + bi$ be any nonzero complex number. It can be shown that $\mathbb{C}$ is a field if there exists a $z_2 = x + yi$ satisfying which of the following equations?

A. $\begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} -a & b \\ -b & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

C. $\begin{bmatrix} -a & b \\ b & -a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

D. $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

11. Which of the following sets is an ordered field?

A. the complex numbers

B. the rational numbers

C. the integers

D. the natural numbers

12. **Use the information below to answer the question that follows.**

Given $z = a + bi$, $\overline{z} = a - bi$ $(a \neq b \neq 0)$, and $\left|z^2\right| = a^2 + b^2$, prove that $z \cdot \overline{z} = \left|z^2\right|$.

| | |
|---|---|
| Step 1 | $z \cdot \overline{z} = (a + bi)(a - bi)$ |
| Step 2 | $(a + bi)(a - bi) = a^2 - abi + bai + b^2$ |
| Step 3 | $a^2 - abi + bai + b^2 = a^2 + b^2$ |
| Step 4 | Therefore $z \cdot \overline{z} = \left|z^2\right|$ |

Which of the following properties is one justification for the simplification made in Step 3?

A. additive inverse property

B. multiplicative inverse property

C. multiplicative identity property

D. modulus property

13. According to the Rational Root Theorem, which of the following is a possible root of $f(x) = 3x^3 + 7x^2 + 11x + 5$?

A. $-5$

B. $-3$

C. $-\frac{3}{5}$

D. $-\frac{1}{5}$

14. If $f(x)$ is a fourth-degree polynomial with real coefficients such that $\dfrac{f(x)}{(x-3)} = q(x) + \dfrac{8}{(x-3)}$, which of the following statements about $f(x)$ must be true?

    A. $f(x)$ has a zero at $x = 3$.

    B. The graph of $y = f(x)$ has a local minimum at $(-3, 8)$.

    C. $f(x)$ has two real roots and two complex roots.

    D. The graph of $y = f(x)$ contains the point $(3, 8)$.

15. If $f(x) = -2x^2 + 8x + 16$, then which of the following is the absolute value of the difference between the zeros of $f(x)$?

    A. 4

    B. $4i$

    C. $4\sqrt{3}$

    D. $4\sqrt{6}$

16. Which of the following are the imaginary parts of the roots of $iz^2 + (2 + i)z + 1$?

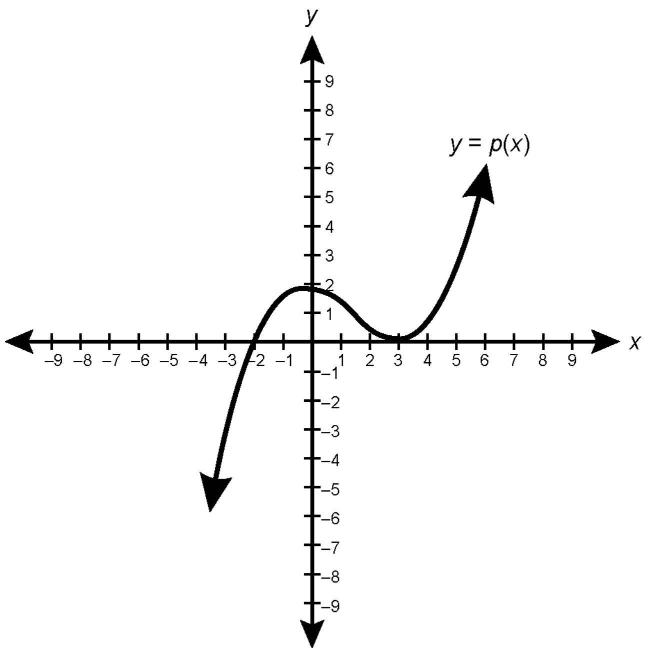
    A. $\dfrac{-1 \pm \sqrt{3}}{2}$

    B. $\dfrac{-2 \pm \sqrt{3}}{2}$

    C. $\dfrac{1 \pm \sqrt{3}}{2}$

    D. $\dfrac{2 \pm \sqrt{3}}{2}$

17. **Use the graph of a polynomial function below to answer the question that follows.**




Which of the following statements about $p(x)$ must be true?

A. $p(x)$ has at least one complex root.

B. $p(x)$ is divisible by $(x - 2)$.

C. $p(x)$ is an odd function.

D. $p(x)$ is divisible by $x^2 - 6x + 9$.

18. Which of the following is a point of intersection of the function $y = 2\sqrt{x}$ and its inverse function?

A. $\left(\frac{1}{4}, 1\right)$

B. (1, 2)

C. (4, 4)

D. $(8, 4\sqrt{2})$

19. Line $\ell$ passes through the points (–7, –6) and (8, 14). What is the *x*-intercept of the line that is perpendicular to line $\ell$ at its *y*-intercept?

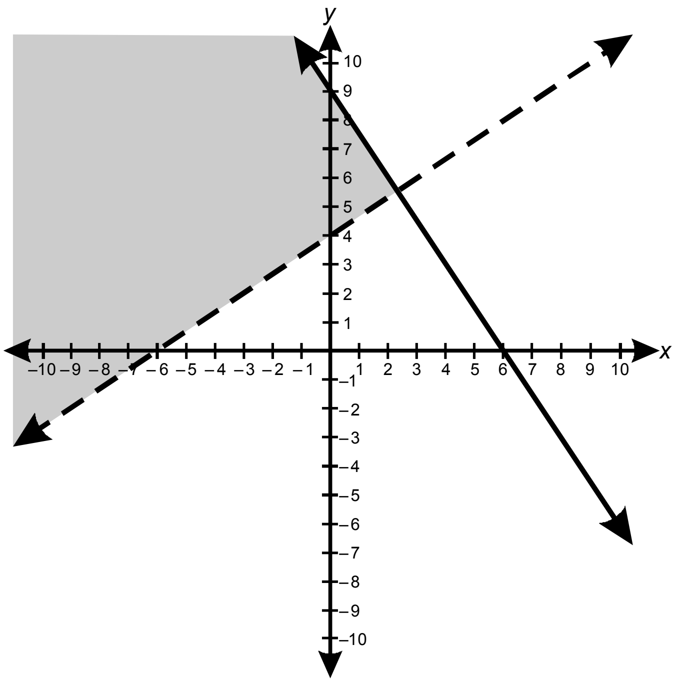
A. $\frac{25}{6}$

B. $\frac{13}{3}$

C. $\frac{40}{9}$

D. $\frac{9}{2}$

20. **Use the graph below to answer the question that follows.**




Which of the following systems of inequalities represents the shaded region above?

A. $3y - 2x \geq 12$
$2y + 3x > 18$

B. $3y - 2x \leq 12$
$2y + 3x > 18$

C. $2x - 3y > -12$
$3x + 2y \leq 18$

D. $2x - 3y < -12$
$3x + 2y \leq 18$

21. Which of the following is an equation for the slant asymptote of $f(x) = \dfrac{x^3 - 1}{x^2}$?

A. $y = x - \dfrac{1}{x^2}$

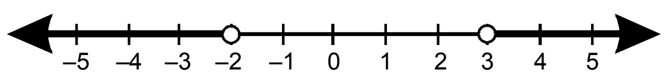
B. $y = x - \dfrac{1}{x}$

C. $y = x - 1$

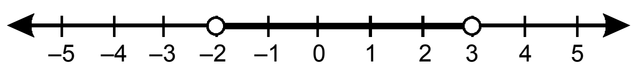
D. $y = x$

22. If $f(x) = x^2 - x - 6$ and $g(x) = \dfrac{\sqrt{x}}{x}$, which of the following number lines represents the domain of $h(x) = g(f(x))$?

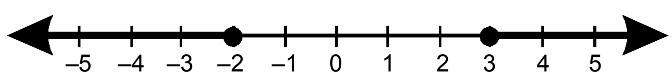
A.



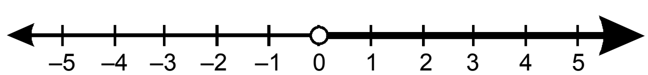

B.




C.




D.



23. If $f(x) = \dfrac{e^{5x} + 6}{2}$ and $g(f(x)) = x$, then which of the following is equivalent to $g(x)$?

A. $\dfrac{\ln(2x - 6)}{5}$

B. $\dfrac{2x - 6}{5}$

C. $\dfrac{\ln(2x - 6)}{e^5}$

D. $\ln \dfrac{2x - 6}{5}$

24. The population of a town is given by the function $P(t) = 8800(1.3)^t$ where $t$ represents the number of years and $0 \le t \le 8$. Which of the following expresses the number of years until the town reaches a population of 18,000 people?

A. $\dfrac{\ln 45 - \ln 22}{\ln 1.3}$

B. $\ln 45 - \ln 22 - \ln 1.3$

C. $\dfrac{\ln 1.3}{\ln 45 - \ln 22}$

D. $\ln \frac{45}{22} - \ln 1.3$

25. Given any two vectors **a** and **b** such that $|\mathbf{a}| = |\mathbf{b}| = 1$, which of the following statements about the inner product, $\mathbf{a} \bullet \mathbf{b}$, must be true?

A. $\mathbf{a} \bullet \mathbf{b} = 1$

B. $-1 \le \mathbf{a} \bullet \mathbf{b} \le 1$

C. $1 \le \mathbf{a} \bullet \mathbf{b} \le \sqrt{2}$

D. $1 \le \mathbf{a} \bullet \mathbf{b} \le 2$

26. Given vectors **a** and **b** such that $|\mathbf{a}| = \frac{10\sqrt{3}}{3}$, $|\mathbf{b}| = 2$,

    and $\mathbf{a} \times \mathbf{b} = 2\sqrt{5}\,\mathbf{i} + 2\sqrt{5}\,\mathbf{j} - 2\sqrt{15}\,\mathbf{k}$, what is the angle

    between **a** and **b**?

    A. 30°

    B. 45°

    C. 60°

    D. 90°

27. **Use the matrix below to answer the question that follows.**

$$\begin{bmatrix} 1 & 2 & 3 \\ -2 & -3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Which of the following matrices has the same determinant as the matrix above?

A. $\begin{bmatrix} -2 & -3 & 2 \\ 1 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 2 & 3 \\ -2 & -3 & 2 \\ 2 & 4 & 4 \end{bmatrix}$

C. $\begin{bmatrix} 2 & 4 & 6 \\ -2 & -3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 2 & 1 & 3 \\ -3 & -2 & 2 \\ 2 & 1 & 1 \end{bmatrix}$

28. Given the equation $\det(A - \lambda I) = 0$, where $A = \begin{bmatrix} 3 & 1 \\ -2 & 6 \end{bmatrix}$ and $I$ is the identity matrix, which of the following is a value of $\lambda$?

A. $-4$

B. $-\frac{1}{3}$

C. 5

D. 20

29. If $A\mathbf{x} = \mathbf{b}$ represents a linear system of equations in three-dimensional space and the $\det(A) \neq 0$, then the solution to this system is a:

A. point.

B. line.

C. two-dimensional plane.

D. three-dimensional plane.

# Constructed-Response Assignments

Read each assignment carefully before you begin your response. Think about how you will organize your response. An erasable notebooklet will be provided at the test center for you to make notes, write an outline, or otherwise prepare your response. For the examination, your final response to each constructed-response assignment must be either:

1) typed into the on-screen response box,
2) written on a response sheet and scanned using the scanner provided at your workstation, or
3) provided using both the on-screen response box (for typed text) and a response sheet (for calculations or drawings) that you will scan using the scanner provided at your workstation.

30. **Complete the exercise that follows.**

Use the principle of mathematical induction to prove the following statement.

$$\frac{1}{1 \bullet 2} + \frac{1}{2 \bullet 3} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

31. **Complete the exercise that follows.**

A cubic function of the form $f(x) = x^3 + c$, where $c$ is a real number, has one zero at $x = 1 + i\sqrt{3}$.

- Find the cubic function; and
- sketch the graph of the function and label any intercepts.

32. **Complete the exercise that follows.**

If vectors **a** = $(a_1, a_2)$ and **b** = $(b_1, b_2)$ are perpendicular, $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, and if we identify vectors with column matrices in the usual manner, then show that the vectors $A$**a** and $A$**b** are perpendicular for all values of $\theta$.

# Annotated Responses to Sample Multiple-Choice Questions for CSET: Mathematics Subtest I

## Number and Quantity

1. **Correct Response: C.** (SMR Code: 1.1) One element is the inverse of another element under an operation if, when the two elements are combined through the operation, the result is the identity element. According to the table, the identity element under operation @ for set $M$ is $b$. Since $a$ @ $c = c$ @ $a = b$, $c$ is the inverse element of $a$.

2. **Correct Response: B.** (SMR Code: 1.1) To simplify the fraction, first reduce $\frac{25}{10}$ by dividing numerator and denominator by the greatest common factor of 5: $\frac{25 \div 5}{10 \div 5} = \frac{5}{2}$. The quotient $\frac{(2)^{10b-8}}{(2)^{5b+2}}$ can be simplified by subtracting the exponents and keeping the same base: $2^{(10b-8)-(5b+2)} = 2^{10b-8-5b-2} = 2^{5b-10}$. The resulting fraction $\frac{5 \bullet (2)^{5b-10}}{2}$ can be further reduced by dividing $(2)^{5b-10}$ by 2 (or $2^1$). Again, keep the base the same and subtract the exponents: $\frac{(2)^{5b-10}}{2} = 2^{5b-10-1} = 2^{5b-11}$. The correct response is $5 \bullet (2)^{5b-11}$.

3. **Correct Response: C.** (SMR Code: 1.1) The ratio $x$:$y$ = 4:5 = $\left(4 \bullet \frac{3}{5}\right):\left(5 \bullet \frac{3}{5}\right) = \left(\frac{12}{5}\right):3$. The ratio $y$:$z$ = 3:8. Therefore, the ratio $x$:$y$:$z$ = $\frac{12}{5}:3:8 = \left(\frac{12}{5} \bullet 5\right):(3 \bullet 5):(8 \bullet 5)$ = 12:15:40. The value of $y$ is $\frac{15}{67}$ of the sum 871, which is 195.

4. **Correct Response: D.** (SMR Code: 1.1 ) To simplify the fraction, multiply $\frac{5-i}{1+7i}$ by $\frac{1-7i}{1-7i}$:

$$\frac{5-i}{1+7i} \bullet \frac{1-7i}{1-7i} = \frac{(5-i)(1-7i)}{(1+7i)(1-7i)} = \frac{5-35i-i+7i^2}{1-49i^2} = \frac{5-36i+7(-1)}{1-49(-1)} = \frac{5-36i-7}{1+49} = \frac{-2-36i}{50} = \frac{-1-18i}{25} =$$

$$-\frac{1}{25} - \frac{18}{25}i$$

5. **Correct Response: C.** (SMR Code: 1.2) To identify all prime numbers less than 200, it is only necessary to eliminate all multiples of each prime number less than (or equal to) $\sqrt{200}$. This is because no composite number less than 200 will have a factor greater than $\sqrt{200}$ without having another factor less than $\sqrt{200}$. Since $\sqrt{200} \approx 14.14$, the person would need to check multiples of 2, 3, 5, 7, 11 and 13.

6. **Correct Response: C.** (SMR Code: 1.2) Each factor of $2^x3^y5^z$ is the product of between 0 and $x$ twos, between 0 and $y$ threes, and between 0 and $z$ fives. Since this yields $(x + 1)$ possible products of twos, $(y + 1)$ possible products of threes and $(z + 1)$ possible products of fives, there are $(x + 1)(y + 1)(z + 1)$ factors.

7. **Correct Response: A.** (SMR Code: 1.2) The proof begins by assuming that $d$ divides $a$ and $b$. The object is to prove that $d$ divides $r$. Since $a = nb + r$, then $r = a - nb$. If $d$ divides $a$ and $b$, then $d$ divides $a - b$. If $d$ divides $a - b$, then $d$ divides $a - 2b$. Likewise, $d$ divides $a - 3b$ and so on. Since $d$ divides $a - nb$, $d$ divides $r$. Thus, the correct response is A.

8. **Correct Response: B.** (SMR Code: 1.2) Since $a = nb + r$, then $r = a - nb$ and any number that divides $a$ and $b$ must also divide $r$. This includes the greatest common divisor of $a$ and $b$, so gcd($a$, $b$) divides $r$. By definition, the greatest common divisor of $a$ and $b$ divides $b$, so gcd($a$, $b$) also divides $b$. Therefore, gcd($a$, $b$) divides both $b$ and $r$. This divisor is either the greatest common divisor of $b$ and $r$, or less than the greatest common divisor of $b$ and $r$, so the following holds: gcd($a$, $b$) $\leq$ gcd($b$, $r$). Similarly, any number that divides both $b$ and $r$ must also divide $a$, so gcd($b$, $r$) divides $a$. Also, gcd($b$, $r$) divides $b$. Therefore, gcd($b$, $r$) divides both $a$ and $b$ and it follows that gcd($b$, $r$) $\leq$ gcd($a$, $b$). Combining the two inequalities shows that gcd($b$, $r$) = gcd($a$, $b$).

## Algebra

9. **Correct Response: A.** (SMR Code: 2.1) A field is a commutative ring with identity in which every nonzero element has an inverse. The set of $3 \times 3$ invertible matrices is not a commutative ring because there exist matrices $A$ and $B$ such that $AB \neq BA$.

10. **Correct Response: D.** (SMR Code: 2.1) Since it is given that $\mathbb{C}$ is a commutative ring, to show that it is a field only requires showing that any nonzero $z_1 = a + bi$ has a multiplicative inverse in $\mathbb{C}$. Let $z_2 = x + yi$ be the multiplicative inverse of $z_1$. Then $z_1 z_2 = 1 + 0i = (a + bi)(x + yi) = ax + ayi + bxi - by = ax - by + (bx + ay)i = 1 + 0i$. The system of equations that results is $ax - by = 1$ and $bx + ay = 0$. The matrix representation of this system is $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

11. **Correct Response: B.** (SMR Code: 2.1) Of the sets given in the response choices, only the complex numbers and the rational numbers are fields. Since the complex numbers are not ordered, response choice B is the correct response.

12. **Correct Response: A.** (SMR Code: 2.1) Note that in the expression on the left-hand side of Step 3, $a^2 - abi + bai + b^2$, the two terms $-abi + bai$ sum to zero, by the additive inverse property.

13. **Correct Response: A.** (SMR Code: 2.2) By the Rational Root Theorem, possible roots for $f(x)$ are factors of 5 divided by factors of 3. These are $\pm\frac{1}{3}$, $\pm\frac{5}{3}$, $\pm 1$, and $\pm 5$. Of the available choices in the response options, A, or $-5$, is the correct response.

14. **Correct Response: D.** (SMR Code: 2.2) Solve the given equation for $f(x)$ to obtain $f(x) = q(x)(x - 3) + 8$. Let $x = 3$, so that $f(3) = q(3)(3 - 3) + 8 \Rightarrow f(3) = 8$. Therefore, the point (3, 8) is on the graph of $y = f(x)$ and the correct response is D.

15. **Correct Response: C.** (SMR Code: 2.2) To find the zeros of $f(x)$, set the function equal to zero and solve for $x$ by using the quadratic formula: $-2x^2 + 8x + 16 = 0 \Rightarrow x = \dfrac{-8 \pm \sqrt{8^2 - 4(-2)(16)}}{2(-2)} \Rightarrow x = \dfrac{-8 \pm \sqrt{192}}{-4}$. Simplifying $\sqrt{192}$ gives $x = \dfrac{-8 \pm 8\sqrt{3}}{-4} = 2 \pm 2\sqrt{3}$. The absolute value of the difference between the zeros is $\left|\left(2 + 2\sqrt{3}\right) - \left(2 - 2\sqrt{3}\right)\right|$, which simplifies to $\left|4\sqrt{3}\right|$ or $4\sqrt{3}$.

16. **Correct Response: D.** (SMR Code: 2.2) Finding the roots of $iz^2 + (2 + i)z + 1$ involves setting the expression equal to 0 and solving for $z$. The quadratic formula gives $z = \dfrac{-(2 + i) \pm \sqrt{(2 + i)^2 - 4(i)(1)}}{2(i)} \Rightarrow$ $z = \dfrac{-2 - i \pm \sqrt{3}}{2i} \Rightarrow z = i - \dfrac{1}{2} \pm \dfrac{i\sqrt{3}}{2}$. Since the roots of the equation are $-\dfrac{1}{2} + \left(1 \pm \dfrac{\sqrt{3}}{2}\right)i$, their imaginary parts are $\dfrac{2 \pm \sqrt{3}}{2}$.

17. **Correct Response: D.** (SMR Code: 2.2) The polynomial $p(x)$ has $x$-intercepts at 3 and $-2$, so $(x - 3)$ and $(x + 2)$ are factors. In fact, $x = 3$ is a double root, since the function is tangent to the $x$-axis at $x = 3$. This means that $(x - 3)^2$, or $(x^2 - 6x + 9)$, is a factor of $p(x)$, so $p(x)$ is divisible by $x^2 - 6x + 9$.

18. **Correct Response: C.** (SMR Code: 2.3) The inverse of $y = 2\sqrt{x}$ is $x = 2\sqrt{y}$, which can be rewritten as $y = \dfrac{x^2}{4}$. However, domain restrictions limit this inverse to only positive $x$-values, so the inverse of $y = 2\sqrt{x}$ is $y = \dfrac{x^2}{4}$, $x \geq 0$. To find the intersection of these two functions, solve $2\sqrt{x} = \dfrac{x^2}{4}$ for $x$ values $\geq 0$. $4x = \dfrac{x^4}{16}$ $\Rightarrow x^4 - 64x = 0 \Rightarrow x(x^3 - 64) = 0$. Hence $x = 0$ or $x = 4$. Since $y = 2\sqrt{x}$, (0, 0) and (4, 4) are points where the two curves intersect. Therefore C is the correct response.

19. **Correct Response: C.** (SMR Code: 2.3) The slope of line $\ell$ is $\dfrac{-6 - 14}{-7 - 8}$, or $\dfrac{4}{3}$. Substituting one of the given points into $y = \frac{4}{3}x + b$ gives an equation of $y = \frac{4}{3}x + \frac{10}{3}$, so the $y$-intercept of $\ell$ is $\left(0, \frac{10}{3}\right)$. Therefore, a line perpendicular to $\ell$ at its $y$-intercept has an equation of $y = -\frac{3}{4}x + \frac{10}{3}$. To find the $x$-intercept of this line, substitute 0 for $y$ and solve for $x$. The $x$-intercept is $\frac{40}{9}$.

20. **Correct Response: D.** (SMR Code: 2.3) The equation of the solid line is $y = -\frac{3}{2}x + 9$, or $2y + 3x = 18$ in standard form. Substituting a point in the shaded region [(0, 6), for example] shows that the inequality is $2y + 3x \leq 18$. The dotted line is given by $3y - 2x = 12$. Substituting (0, 6) gives the inequality $3y - 2x > 12$ (the dotted line indicates that points on the boundary line are not solutions to the inequality). This inequality can be rewritten as $-3y + 2x < -12$, so response choice D is correct.

21. **Correct Response: D.** (SMR Code: 2.3) Note that $f(x) = \frac{x^3 - 1}{x^2} = x - \frac{1}{x^2}$. As $x$ approaches positive or negative infinity, $\frac{1}{x^2}$ approaches 0 and f($x$) approaches $x$. Therefore, the asymptote of $f(x)$ is the line $y = x$.

22. **Correct Response: A.** (SMR Code: 2.3) The composition of $g(f(x)) = h(x) = \frac{\sqrt{x^2 - x - 6}}{x^2 - x - 6} = \frac{1}{\sqrt{x^2 - x - 6}}$.

The domain of $h(x)$ is such that $x^2 - x - 6 > 0$. Since $x^2 - x - 6 = (x + 2)(x - 3)$, $x = -2$ and $x = 3$ are zeros of this expression. This expression will be positive when both $(x + 2)$ and $(x - 3)$ are positive or when both $(x + 2)$ and $(x - 3)$ are negative. This occurs when $x < -2$ or $x > 3$, as shown in the graph in response A.

23. **Correct Response: A.** (SMR Code: 2.3) If $g(f(x)) = x$, then $f(x)$ and $g(x)$ are inverses of each other. If $f(x) = \frac{e^{5x} + 6}{2}$, then $x = \frac{e^{5g(x)} + 6}{2}$. This equation can be rewritten as $2x - 6 = e^{5g(x)}$, and taking the natural log of both sides yields $\ln(2x - 6) = \ln e^{5g(x)} \Rightarrow 5g(x)(\ln e) = \ln(2x - 6)$. Then, since $\ln e = 1$, $g(x) = \frac{1}{5}\ln(2x - 6)$.

24. **Correct Response: A.** (SMR Code: 2.3) To find when the population equals 18,000 people, solve $P(t)$ for $t$. If $P(t) = 18{,}000$ and $P(t) = 8800(1.3)^t$, then $18{,}000 = 8800(1.3)^t \Rightarrow \frac{18000}{8800} = (1.3)^t \Rightarrow \frac{45}{22} = (1.3)^t$. Therefore, $\ln\frac{45}{22} = \ln(1.3)^t$ and $t = \frac{\ln 45 - \ln 22}{\ln 1.3}$.

25. **Correct Response: B.** (SMR Code: 2.4) The dot product of vectors **a** and **b** can be written as $\mathbf{a} \bullet \mathbf{b} = |\,\mathbf{a}\,||\,\mathbf{b}\,| \cos \theta$, where $\theta$ is the angle between the two vectors. Given that **a** and **b** are unit vectors, $\mathbf{a} \bullet \mathbf{b} = \cos \theta$. Since $-1 \leq \cos \theta \leq 1$, $-1 \leq \mathbf{a} \bullet \mathbf{b} \leq 1$.

26. **Correct Response: C.** (SMR Code: 2.4) The magnitude of the cross product of two vectors is given by $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$. Since $|\mathbf{a}|$ and $|\mathbf{b}|$ are both given and $|\mathbf{a} \times \mathbf{b}| = \sqrt{(2\sqrt{5})^2 + (2\sqrt{5})^2 + (-2\sqrt{15})^2} =$ 10, it follows that $= \left(\frac{10\sqrt{3}}{3}\right)(2)\sin\theta = 10 \Rightarrow \frac{\sqrt{3}}{2} = \sin\theta$. Since $\arcsin\frac{\sqrt{3}}{2} = 60°$, the angle between the two vectors is 60°.

27. **Correct Response: B.** (SMR Code: 2.4) The matrix in each response choice is obtained from the given matrix by an elementary row operation. The matrix in response choice B is obtained from the given matrix by adding row 1 to row 3. This elementary row operation does not change the value of the determinant of the matrix.

28. **Correct Response: C.** (SMR Code: 2.4) Note that $(A - \lambda I) = \begin{bmatrix} 3 & 1 \\ -2 & 6 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 3-\lambda & 1 \\ -2 & 6-\lambda \end{bmatrix}$. Therefore, $\det(A - \lambda I) = \det\begin{bmatrix} 3-\lambda & 1 \\ -2 & 6-\lambda \end{bmatrix} = (3-\lambda)(6-\lambda) - (-2) = \lambda^2 - 9\lambda + 20$. Since $\det(A - \lambda I) = 0$, then $0 = \lambda^2 - 9\lambda + 20$. Solving for $\lambda$ gives $0 = (\lambda - 5)(\lambda - 4)$ and $\lambda = 4$ or $\lambda = 5$. Of the available responses, C is correct.

29. **Correct Response: A.** (SMR Code: 2.4) Since $\det(A) \neq 0$, then $A^{-1}$ exists and is unique. Thus, $A^{-1}A\mathbf{x} = A^{-1}\mathbf{b} \Rightarrow I\mathbf{x} = A^{-1}\mathbf{b} \Rightarrow \mathbf{x} = A^{-1}\mathbf{b}$ is a unique solution to the linear system of equations given by $A\mathbf{x} = \mathbf{b}$. Since this is a linear system in three-dimensional space, $\mathbf{x}$ represents a point in space.

# Examples of Strong Responses to Sample Constructed-Response Questions for CSET: Mathematics Subtest I

## Number and Quantity

**Question #30 (Score Point 4 Response)**

Initial Step: show the statement is true for $n = 1$.

left-hand side: $\frac{1}{1 \bullet 2} + \frac{1}{2 \bullet 3} + \ldots + \frac{1}{n(n+1)} = \frac{1}{1(1+1)} = \frac{1}{2}$

right-hand side: $\frac{n}{n+1} = \frac{1}{1+1} = \frac{1}{2}$

$\therefore$ the statement is true for $n = 1$.

Inductive Step: assume the statement is true for $n = K$, and then prove that it is true for $n = K + 1$.

So assume $\frac{1}{1 \bullet 2} + \frac{1}{2 \bullet 3} + \ldots + \frac{1}{K(K+1)} = \frac{K}{K+1}$ (1)

Now want to show that $\frac{1}{1 \bullet 2} + \frac{1}{2 \bullet 3} + \ldots + \frac{1}{(K+1)(K+2)} = \frac{K+1}{K+2}$

Adding $\frac{1}{(K+1)(K+2)}$ to each side of equation (1) gives:

$$\frac{1}{1 \bullet 2} + \frac{1}{2 \bullet 3} + \ldots + \frac{1}{K(K+1)} + \frac{1}{(K+1)(K+2)} = \frac{K}{K+1} + \frac{1}{(K+1)(K+2)} = \frac{K(K+2)+1}{(K+1)(K+2)}$$

$$= \frac{K^2 + 2K + 1}{(K+1)(K+2)}$$

$$= \frac{(K+1)(K+1)}{(K+1)(K+2)}$$

$$= \frac{(K+1)}{(K+2)}$$

*continued on next page*

**Question #30 (Score Point 4 Response)** ***continued***

So the statement is true for $n = k + 1$.

Therefore, since both the initial and inductive steps have been completed, by induction, the statement is true for all natural numbers $n$.

## Algebra

**Question #31 (Score Point 4 Response)**

If $f(x) = x^3 + c$, where $c$ is a real number, and $f(x)$ has a zero at $x = 1 + i\sqrt{3}$, then its conjugate $1 - i\sqrt{3}$ must also be a root.

Thus $x - \left(1 + i\sqrt{3}\right)$ and $x - \left(1 - i\sqrt{3}\right)$ both are factors of $f(x) = x^3 + c$, so their product is a quadratic factor of $f(x) = x^3 + c$.

$$\left[x - \left(1 + i\sqrt{3}\right)\right]\left[x - \left(1 - i\sqrt{3}\right)\right] = \left[x - 1 - i\sqrt{3}\right]\left[x - 1 + i\sqrt{3}\right]$$

$$= x^2 - x + xi\sqrt{3} - x + 1 - i\sqrt{3} - xi\sqrt{3} + i\sqrt{3} - 3i^2$$

$$= x^2 - 2x + 4 \text{ is a quadratic factor of } f(x) = x^3 + c$$

Let $a = \sqrt[3]{c}$, so $f(x) = x^3 + a^3$.

Now factor $x^3 + a^3$ as the sum of 2 cubes:

$$f(x) = x^3 + a^3 = (x + a)(x^2 - ax + a^2)$$

$$= (x + a)(x^2 - 2x + 4), \text{ since } x^2 - 2x + 4 \text{ is a factor of } x^3 + c.$$

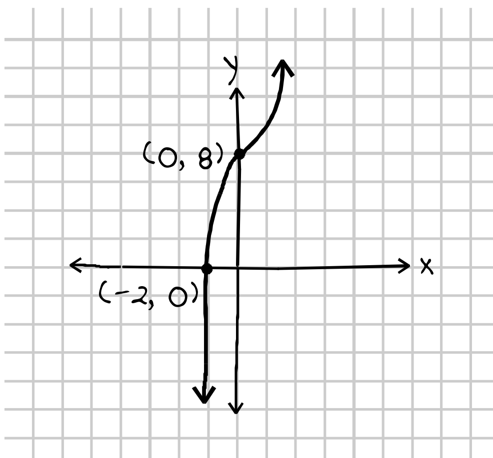
*continued on next page*

**Question #31 (Score Point 4 Response)** ***continued***

Thus $a = 2 \Rightarrow f(x) = x^3 + a^3 = x^3 + 2^3 = x^3 + 8$

| x | f(x) |
|---|---|
| 0 | 8 |
| -2 | 0 |

The graph of $f(x) = x^3 + 8$ is the graph of $f(x) = x^3$ shifted up 8 units.




Intercepts are (0, 8) and (-2, 0).

**Question #32 (Score Point 4 Response)**

The dot product $\vec{a} \bullet \vec{b} = a_1b_1 + a_2b_2 = 0$ if and only if the vectors $\vec{a} = (a_1, a_2)$ and $\vec{b} = (b_1, b_2)$ are perpendicular. Want to show that $A\vec{a} \bullet B\vec{b} = 0$.

$$A\vec{a} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} a_1\cos\theta - a_2\sin\theta \\ a_1\sin\theta + a_2\cos\theta \end{bmatrix}$$

$$A\vec{b} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} b_1\cos\theta - b_2\sin\theta \\ b_1\sin\theta + b_2\cos\theta \end{bmatrix}$$

$$\begin{aligned} A\vec{a} \bullet A\vec{b} &= (a_1\cos\theta - a_2\sin\theta, a_1\sin\theta + a_2\cos\theta) \bullet (b_1\cos\theta - b_2\sin\theta, b_1\sin\theta + b_2\cos\theta) \\ &= a_1b_1\cos^2\theta - a_1b_2\cos\theta\sin\theta - a_2b_1\cos\theta\sin\theta + a_2b_2\sin^2\theta + \\ &\qquad a_1b_1\sin^2\theta + a_1b_2\cos\theta\sin\theta + a_2b_1\cos\theta\sin\theta + a_2b_2\cos^2\theta \\ &= a_1b_1\cos^2\theta + a_1b_1\sin^2\theta + a_2b_2\sin^2\theta + a_2b_2\cos^2\theta \\ &= a_1b_1(\cos^2\theta + \sin^2\theta) + a_2b_2(\cos^2\theta + \sin^2\theta) \end{aligned}$$

Since $\cos^2\theta + \sin^2\theta = 1$, $A\vec{a} \bullet A\vec{b} = a_1b_1 + a_2b_2 = 0$ (since $\vec{a}$ and $\vec{b}$ are perpendicular).

Hence the vectors $A\vec{a}$ and $A\vec{b}$ are perpendicular for all $\theta$.

# Scoring Information for CSET: Mathematics Subtest I

Responses to the multiple-choice questions are scored electronically. Scores are based on the number of questions answered correctly. There is no penalty for guessing.

There are three constructed-response questions in Subtest I of CSET: Mathematics. Each of these constructed-response questions is designed so that a response can be completed within a short amount of time—approximately 10–15 minutes. Responses to constructed-response questions are scored by qualified California educators using focused holistic scoring. Scorers will judge the overall effectiveness of your responses while focusing on the performance characteristics that have been identified as important for this subtest (see below). Each response will be assigned a score based on an approved scoring scale (see page 29).

Your performance on the subtest will be evaluated against a standard determined by the Commission on Teacher Credentialing based on professional judgments and recommendations of California educators.

## Performance Characteristics for CSET: Mathematics Subtest I

The following performance characteristics will guide the scoring of responses to the constructed-response questions on CSET: Mathematics Subtest I.

| | |
|---|---|
| **PURPOSE** | The extent to which the response addresses the constructed-response assignment's charge in relation to relevant CSET subject matter requirements. |
| **SUBJECT MATTER KNOWLEDGE** | The application of accurate subject matter knowledge as described in the relevant CSET subject matter requirements. |
| **SUPPORT** | The appropriateness and quality of the supporting evidence in relation to relevant CSET subject matter requirements. |
| **DEPTH AND BREADTH OF UNDERSTANDING** | The degree to which the response demonstrates understanding of the relevant CSET subject matter requirements. |

## Scoring Scale for CSET: Mathematics Subtest I

Scores will be assigned to each response to the constructed-response questions on CSET: Mathematics Subtest I according to the following scoring scale.

| SCORE POINT | SCORE POINT DESCRIPTION |
|---|---|
| 4 | **The "4" response reflects a thorough command of the relevant knowledge and skills as defined in the subject matter requirements for CSET: Mathematics.**<br>• The purpose of the assignment is fully achieved.<br>• There is a substantial and accurate application of relevant subject matter knowledge.<br>• The supporting evidence is sound; there are high-quality, relevant examples.<br>• The response reflects a comprehensive understanding of the assignment. |
| 3 | **The "3" response reflects a general command of the relevant knowledge and skills as defined in the subject matter requirements for CSET: Mathematics.**<br>• The purpose of the assignment is largely achieved.<br>• There is a largely accurate application of relevant subject matter knowledge.<br>• The supporting evidence is adequate; there are some acceptable, relevant examples.<br>• The response reflects an adequate understanding of the assignment. |
| 2 | **The "2" response reflects a limited command of the relevant knowledge and skills as defined in the subject matter requirements for CSET: Mathematics.**<br>• The purpose of the assignment is partially achieved.<br>• There is limited accurate application of relevant subject matter knowledge.<br>• The supporting evidence is limited; there are few relevant examples.<br>• The response reflects a limited understanding of the assignment. |
| 1 | **The "1" response reflects little or no command of the relevant knowledge and skills as defined in the subject matter requirements for CSET: Mathematics.**<br>• The purpose of the assignment is not achieved.<br>• There is little or no accurate application of relevant subject matter knowledge.<br>• The supporting evidence is weak; there are no or few relevant examples.<br>• The response reflects little or no understanding of the assignment. |
| U | **The "U" (Unscorable) is assigned to a response that is unrelated to the assignment, illegible, primarily in a language other than English, or does not contain a sufficient amount of original work to score.** |
| B | **The "B" (Blank) is assigned to a response that is blank.** |