# CSET®

## California Subject Examinations for Teachers®

# TEST GUIDE

# MATHEMATICS
# SUBTEST II

## Sample Questions and Responses and Scoring Information

# Sample Test Questions for CSET: Mathematics Subtest II

Below is a set of multiple-choice questions and constructed-response questions that are similar to the questions you will see on Subtest II of CSET: Mathematics.  Please note that, as on the actual test form, approximately one third of the multiple-choice questions in this test guide are more complex questions that require 2–3 minutes each to complete.  You are encouraged to respond to the questions without looking at the responses provided in the next section.  Record your responses on a sheet of paper and compare them with the provided responses.

A calculator will be needed and **will be allowed only for Mathematics Subtest II: Geometry; Probability and Statistics.**  You must bring your own graphing calculator to the test administration, and it must be one of the approved models listed on the California Educator Credentialing Examinations website.  Since the approved calculator brands and models are subject to change, the list of approved graphing calculators will be updated as necessary.  Test administration staff will clear the memory of your calculator before and after the test.  Be sure you back up the memory on your calculator, including applications, to an external device before arriving at the test site.

1.  **Use the diagram below to answer the question that follows.**
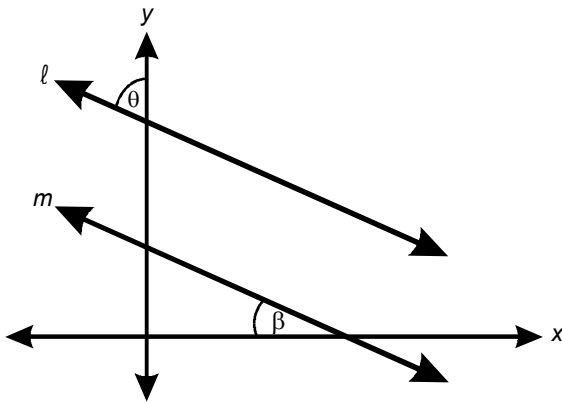


In the diagram above, $\theta$ and $\beta$ are complementary angles and lines $\ell$ and $m$ are given by $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$, respectively.  Which of the following must be true?

A.  $a_1c_1 = a_2c_2$

B.  $b_1c_2 = b_2c_1$

C.  $a_1b_1 = a_2b_2$

D.  $a_1b_2 = a_2b_1$

**Use the incomplete proof below to answer the two questions that follow.**

**Given:** Quadrilateral *ABCD* is a parallelogram.

**Prove:** $\overline{AB} \cong \overline{DC}$; $\overline{BC} \cong \overline{AD}$
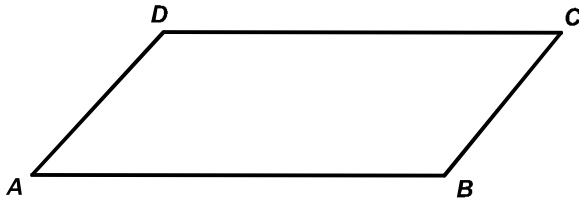


| Statement | Reason |
|---|---|
| 1. Draw $\overline{BD}$ | 1. Between any two points there exists a line. |
| 2. $\overline{BD} \cong \overline{BD}$ | 2. Reflexive property of equality. |
| 3. | 3. |
| 4. $\triangle ABD \cong \triangle CDB$ | 4. |
| 5. $\overline{AB} \cong \overline{CD}$; $\overline{AD} \cong \overline{BC}$ | 5. Corresponding parts of congruent triangles are congruent. |

2. Which of the following is the missing statement and reason for line 3?

A.

| Statement | Reason |
|---|---|
| 3. $\angle ADC \cong \angle CBA$; $\angle DAB \cong \angle DCB$ | 3. Definition of a parallelogram |

B.

| Statement | Reason |
|---|---|
| 3. $\angle ABD \cong \angle BDC$; $\angle ADB \cong \angle CBD$ | 3. If two lines are parallel, alternate interior angles are congruent. |

C.

| Statement | Reason |
|---|---|
| 3. $\triangle ADC \cong \triangle CBA$ | 3. SSS |

D.

| Statement | Reason |
|---|---|
| 3. $\triangle ACD \cong \triangle CAB$ | 3. SAS |

3. Which of the following is the missing reason for statement 4?

A. SAS

B. AAA

C. SSS

D. ASA

4.  Which of the following can be proved without using Euclid's parallel postulate?

    A.  The sum of the angles in a triangle is 180°.

    B.  Vertical angles are congruent.

    C.  Opposite sides of a parallelogram are congruent.

    D.  If two parallel lines are intersected by a transversal, alternate interior angles are congruent.

5.  **Use the statement below to answer the question that follows.**

    > If a transversal intersects two parallel lines, then the alternate interior angles are congruent.

    If the above statement is false, which of the following is also false?

    A.  Supplementary angles sum to 180°.

    B.  Vertical angles are congruent.

    C.  The base angles of an isosceles triangle are congruent.

    D.  The angle sum of every triangle is 180°.

6. **Use the diagram below to answer the question that follows.**
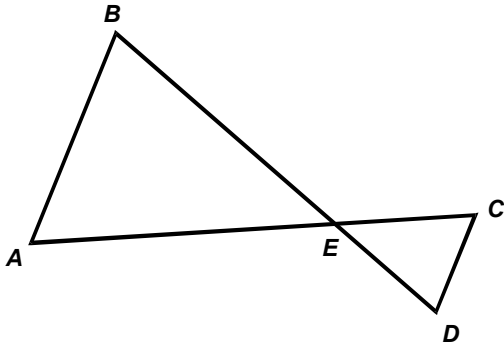


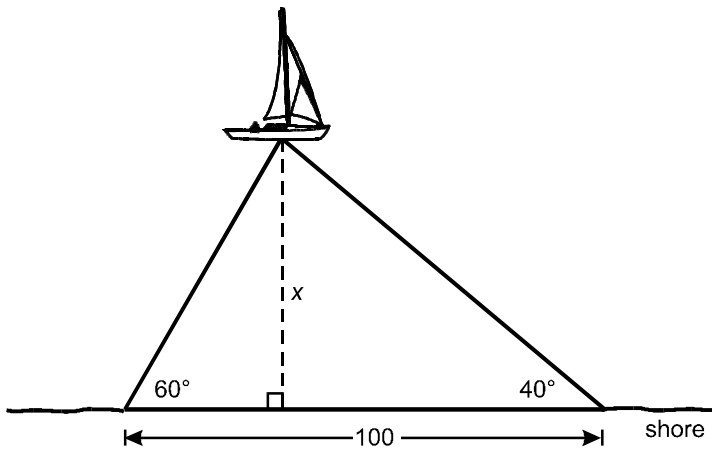If $\overline{AB}$ is parallel to $\overline{CD}$, which of the following is true?

A.   $AC = BD$

B.   $\dfrac{EC}{BE} = \dfrac{CD}{AB}$

C.   $ED = EC$

D.   $\dfrac{ED}{BE} = \dfrac{EC}{AE}$

7. **Use the diagram below to answer the question that follows.**
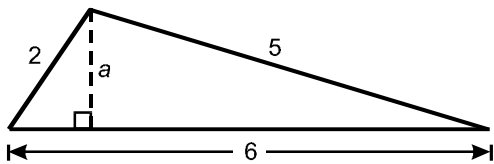


Given that $a$ is an altitude of $\triangle ABC$, what is the length of $a$?

A.   $\sqrt{7}$

B.   $\sqrt{3}$

C.   $\dfrac{\sqrt{39}}{4}$

D.   $\dfrac{\sqrt{5}}{2}$

8.  **Use the diagram below to answer the question that follows.**



A boat is seen in the ocean. The angle and distance measurements in the diagram above are made from the shore. Which of the following equations relating the distance, $x$, of the boat from the shore and the given measurements is true?

A.  $\dfrac{x}{\sin 60°} + \dfrac{x}{\sin 40°} = 100$

B.  $\dfrac{x}{\cos 60°} + \dfrac{x}{\sin 40°} = 100$

C.  $\dfrac{x}{\cos 60°} + \dfrac{x}{\cos 40°} = 100$

D.  $\dfrac{x}{\tan 60°} + \dfrac{x}{\tan 40°} = 100$

9. An artist has been commissioned to build a statue equidistant from the three sides of a triangular park. If the artist has a scale drawing of the triangular park, which of the following constructions could the artist use in determining the location of the statue?

    A. bisecting a given angle

    B. bisecting a given line segment

    C. constructing an angle congruent to a given angle

    D. constructing a segment congruent to a given segment

10. A regular hexagon has a perimeter of 72 cm. Which of the following is the approximate area of the hexagon in cm²?

    A. 144

    B. 339

    C. 374

    D. 452

11. $S(4, 0)$ and $T(-2, 0)$ are two fixed points in a coordinate axis system, and point $P$ has coordinates $(x, y)$. If the length of $\overline{PS}$ is twice the length of $\overline{PT}$, which of the following correctly describes the locus of points satisfying these conditions?

    A. an ellipse with foci $(-2, 0)$ and $(4, 0)$

    B. an ellipse with foci $(-4, 0)$ and $(2, 0)$

    C. a circle with center $(2, 0)$

    D. a circle with center $(-4, 0)$

12. Plane $\alpha$ is perpendicular to plane $\beta$. The two planes intersect in $m$. If line $k$ is perpendicular to $\alpha$ and skew to $m$, then line $k$ is:

    A. parallel to $\beta$.

    B. contained in $\beta$.

    C. tangent to $\beta$.

    D. perpendicular to $\beta$.

13. **Use the diagram below to answer the question that follows.**
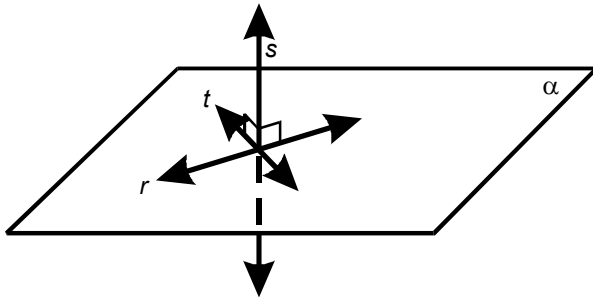


    If line $s$ is perpendicular to lines $r$ and $t$ and both $r$ and $t$ lie in plane $\alpha$, which of the following is true?

    A. Line $s$ is perpendicular to any line in plane $\alpha$ that passes through the intersection of $r$ and $t$.

    B. Lines $r$ and $t$ are perpendicular to each other.

    C. Lines $s$ and $t$ define a plane that forms an acute dihedral angle with plane $\alpha$.

    D. Lines $r$, $s$, and $t$ are coplanar.

14. A cylindrical can is 10 inches in height and has a surface area of about 245 square inches, including the top and bottom. Which of the following is the approximate volume of the can in cubic inches?

   A. 283

   B. 385

   C. 583

   D. 785

15. A sphere of radius $r$ is cut into slices by planes passing through the center of the sphere. Which of the following expressions gives the total surface area of the slices in terms of $n$, the number of cuts?

   A. $\frac{4}{3}\pi r^2 + 2n\pi r^2$

   B. $\frac{4}{3}\pi r^2 + n\pi r^2$

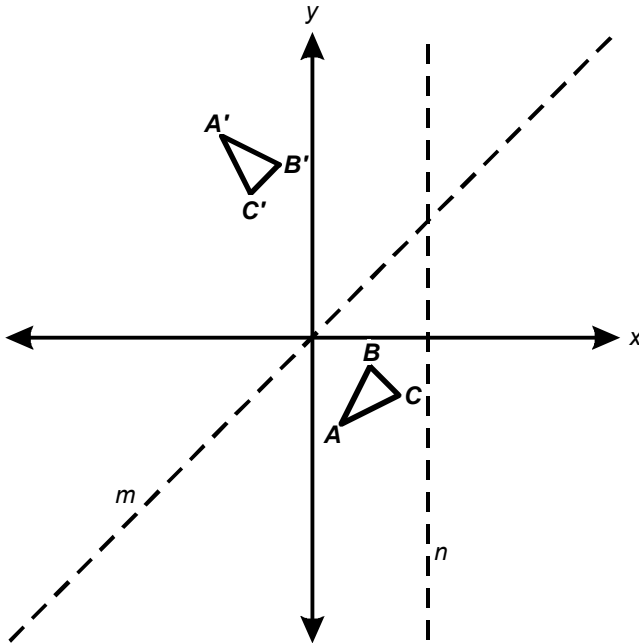   C. $4\pi r^2 + n\pi r^2$

   D. $4\pi r^2 + 2n\pi r^2$

16. **Use the diagram below to answer the question that follows.**
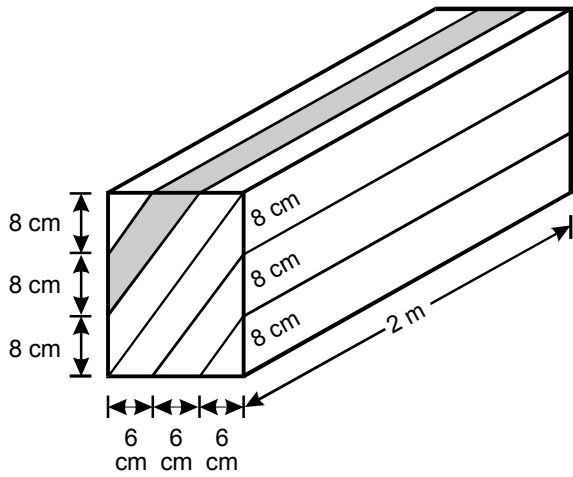


A beam of wood in the shape of a rectangular prism 2 meters long is cut into 6 pieces, as shown in the diagram above. What is the volume of the shaded piece?

A.   14,400 cm³

B.   14,600 cm³

C.   14,800 cm³

D.   15,000 cm³

17. **Use the diagram below to answer the question that follows.**



The diagram above shows the transformation of $\triangle ABC$ under the composition $R_2 R_1$, where $R_1$ is a reflection over line $n$, followed by a reflection $R_2$ over line $m$. Transforming $\triangle ABC$ under the composition $R_1 R_2$ would demonstrate that:

A. the set of reflections does not include inverse elements.

B. the composition of transformations is not commutative.

C. the composition of two reflections is not an isometry.

D. the composition of transformations is not associative.

18.  **Use the diagrams below to answer the question that follows.**
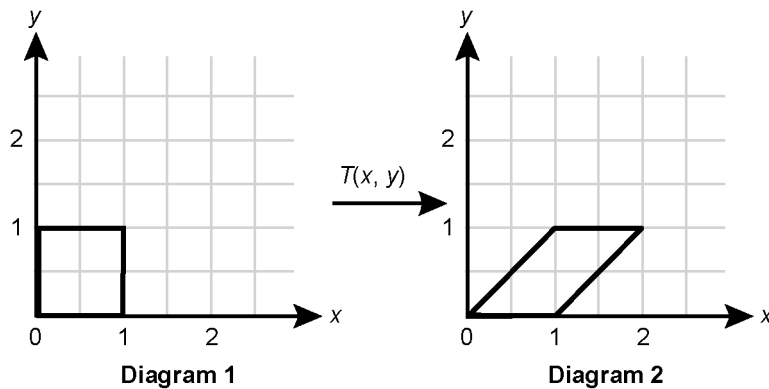


Diagram 1          Diagram 2

Transformation $T(x, y)$ maps the square in Diagram 1 to the parallelogram in Diagram 2.  Which of the following could be $T$?

A.  $T(x, y) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

B.  $T(x, y) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

C.  $T(x, y) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

D.  $T(x, y) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

19. Lines *m* and *k* are perpendicular. Plane figure *ABC* is first reflected over line *m,* and then reflected over line *k.* This transformation is equivalent to which of the following transformations?

   A. 90° rotation

   B. 180° rotation

   C. glide reflection

   D. reflection over the line bisecting the right angle

20. A hexagon undergoes a coordinate transformation given by

   $T:(x, y) \rightarrow (3x, \frac{1}{2}y)$. What is the ratio of the area of the original

   polygon to the area of the transformed hexagon?

   A. $\frac{4}{9}$

   B. $\frac{2}{3}$

   C. $\frac{3}{2}$

   D. $\frac{9}{4}$

21. A test contains 50 multiple-choice questions and each question has five possible answers. If a student answers every question, how many different ways can the student answer the questions on the test?

   A. $5^{50}$

   B. $50^{5}$

   C. $5! \times 50$

   D. $5 \times 50!$

22. A collection of 22 laptops includes 6 defective laptops. If a sample of 3 laptops is randomly chosen from the collection, what is the probability that at least one laptop in the sample will be defective?

    A. 0.136

    B. 0.273

    C. 0.364

    D. 0.636

23. The volume of liquid in soda cans is normally distributed with a mean of 12 fl. oz. and a standard deviation of 0.05 fl. oz. What is the approximate percentage of cans of this brand of soda that contain less than 11.9 fl. oz.?

    A. 0.5%

    B. 1%

    C. 2.5%

    D. 5%

24. If a normally distributed data set has a mean of 40.0 and a standard deviation of 1.5, in which of the following graphs does the shaded area represent the probability that a given member of the set is greater than or equal to 39 and less than or equal to 42?

A.
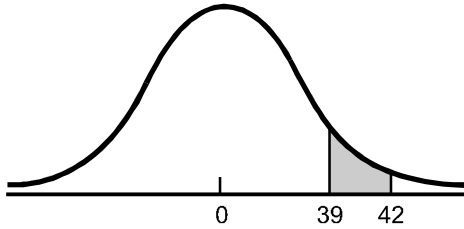


B.
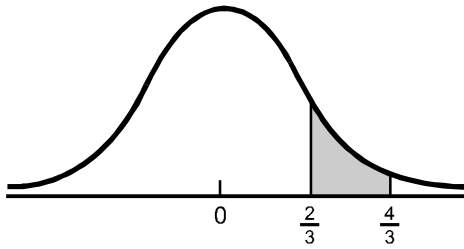


C.
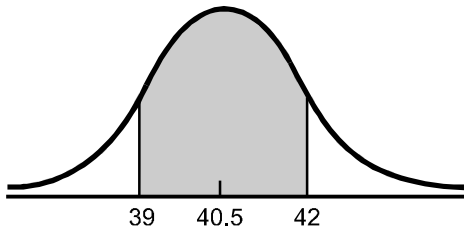


D.
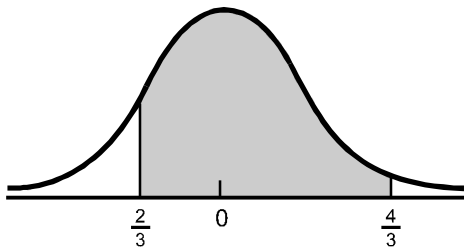
25. A scientist who has been weighing birds collected from the wild discovers that the laboratory scale has been reading out weights that are 1.5 ounces heavier than the birds' actual weights. Which of the following descriptive statistics is likely to be most affected by this error?

    A. skewness

    B. range

    C. variance

    D. quartiles

26. If the standard deviation of the set {4, 9, $x$} is $\sqrt{\frac{14}{3}}$, which of the following is a possible value of $x$?

    A. 11

    B. 8

    C. 7

    D. 2

27. If the regression line for a data set is $y = -2.7x + 4.8$, which of the following is the most reasonable value for the correlation coefficient of the data set?

    A. −2.7

    B. −0.8

    C. 4.8

    D. 0.7

# Constructed-Response Assignments

Read each assignment carefully before you begin your response.  Think about how you will organize your response.  An erasable notebooklet will be provided at the test center for you to make notes, write an outline, or otherwise prepare your response.  For the examination, your final responses to each constructed-response assignment must be either:

1)  typed into the on-screen response box,
2)  written on a response sheet and scanned using the scanner provided at your workstation, or
3)  provided using both the on-screen response box (for typed text) and a response sheet (for calculations or drawings) that you will scan using the scanner provided at your workstation.

28. **Use the diagram below to complete the exercise that follows.**
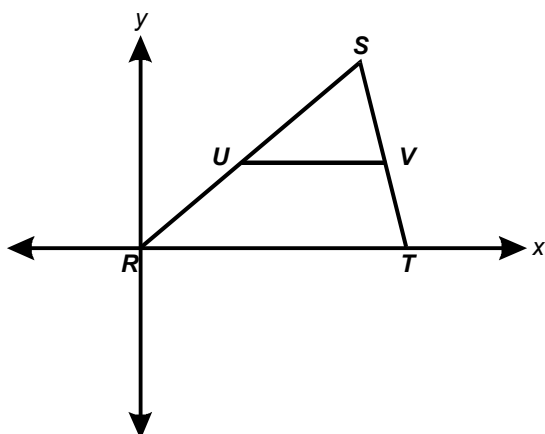


Use techniques of coordinate geometry to prove that the segment joining the midpoints of two sides of a triangle is parallel to the third side of the triangle and one half its length.

29. **Complete the exercise that follows.**

    Find the equation of the curve traced by a point that moves so the sum of its distances to the points (0, 0) and (0, 4) is 12.

30. **Complete the exercise that follows.**

    Housing units in U.S. suburban areas in 1999 were distributed as follows:  69.1% owner occupied, 24.6% renter occupied, and 6.3% vacant.  A researcher, wishing to determine whether this distribution is currently the same, takes a random sample of 500 current housing units in U.S. suburban areas and obtains the following data: 375 owner-occupied units, 100 renter-occupied units, and 25 vacant units.

    *   State the null hypothesis that the researcher should use to determine if these data suggest that the distribution of current year-round housing differs from the 1999 distribution.

    *   Based on the 1999 distribution, determine the number of housing units, out of a sample of 500, expected to be owner occupied, renter occupied, and vacant.

    *   Determine the value of the chi-square ($\chi^2$) test statistic for these sample data.

    *   The table below gives the probability corresponding to given values on a $\chi^2$-distribution with two degrees of freedom.  Use the table to determine if, at the 5% significance level, the data suggest that the distribution of current housing units in U.S. suburban areas differs from the 1999 distribution.

        | **Probability** | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
        |---|---|---|---|---|---|
        | $\chi^2$ | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 |

# Annotated Responses to Sample Multiple-Choice Questions for CSET: Mathematics Subtest II

## Geometry

1. **Correct Response: D.** (SMR Code: 3.1) Since $\theta$ and $\beta$ are complementary angles, the third angle in the

   right triangle in the diagram equals $\theta$. The angle vertical to this third angle $\theta$ also equals $\theta$. Therefore,

   since the alternate exterior angles of lines $\ell$ and $m$ both equal $\theta$, lines $\ell$ and $m$ are parallel. The equations

   of lines $\ell$ and $m$ can be converted to slope-intercept form: $a_1 x + b_1 y = c_1 \Rightarrow y = \dfrac{-a_1}{b_1}x + \dfrac{c_1}{b_1}$ and

   $a_2 x + b_2 y = c_2 \Rightarrow y = \dfrac{-a_2}{b_2}x + \dfrac{c_2}{b_2}$. Since $\ell$ and $m$ are parallel, $\dfrac{-a_1}{b_1} = \dfrac{-a_2}{b_2}$, so $a_1 b_2 = a_2 b_1$.

2. **Correct Response: B.** (SMR Code: 3.1) $\overline{BD}$ is a transversal to both pairs of parallel lines: $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ and $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$. Therefore, $\angle ABD \cong \angle BDC$ and $\angle ADB \cong \angle CBD$, since they are alternate interior angles formed by parallel lines cut by a transversal.

3. **Correct Response: D.** (SMR Code: 3.1) In the two triangles, $\angle ABD \cong \angle BDC$ and $\angle ADB \cong \angle CBD$ are corresponding angles and $\overline{BD} \cong \overline{BD}$ as noted in line 2 of the table. Thus, by angle-side-angle congruence (ASA), $\triangle ABD \cong \triangle CDB$.

4. **Correct Response: B.** (SMR Code: 3.1) Vertical angles are two angles whose sides form two pairs of opposite rays. The proof of the congruence of vertical angles depends primarily on the angle addition postulate and does not require the use of properties of parallel lines that are a result of Euclid's parallel postulate.

5. **Correct Response: D.** (SMR Code: 3.1) In Euclidean geometry, if two parallel lines are cut by a transversal, then the alternate interior angles formed are congruent. This statement is used to prove that the angles of every triangle sum to 180°. If the given statement is false, then the sum of angles in a triangle is not necessarily 180°, as is the case in non-Euclidean geometries.

6. **Correct Response: D.** (SMR Code: 3.1) If $\overline{AB}$ is parallel to $\overline{CD}$, then $\angle A \cong \angle C$ and $\angle B \cong \angle D$, since

   they are alternate interior angles. Then by AA, $\triangle ABE \sim \triangle CDE$, so the ratios between corresponding

   pairs of sides are equal. Hence $\dfrac{ED}{BE} = \dfrac{EC}{AE}$.

7. **Correct Response: C.** (SMR Code: 3.1) Let the base of the right triangle formed by altitude $a$ and side

   of length 2 be $x$. Thus, the base of the right triangle formed by altitude $a$ and side of length 5 is $6 - x$.

   By the Pythagorean theorem, $x^2 + a^2 = 2^2$ and $(6 - x)^2 + a^2 = 5^2$. Solve each equation for $a^2$ and since

   $a^2 = a^2 \Rightarrow 4 - x^2 = 12x - x^2 - 11 \Rightarrow 15 = 12x \Rightarrow \dfrac{5}{4} = x$. To find the length of $a$, use the Pythagorean

   theorem and the value for $x$ to obtain $a = \sqrt{2^2 - \left(\dfrac{5}{4}\right)^2} = \dfrac{\sqrt{39}}{4}$.

8. **Correct Response: D.** (SMR Code: 3.1) Let the base of the right triangle formed by altitude $x$ and the $60°$ angle be $a$. Let the base of the right triangle formed by altitude $x$ and the $40°$ angle be $b$. Note that the $\tan 60° = \frac{x}{a} \Rightarrow a = \frac{x}{\tan 60°}$. Similarly, $\tan 40° = \frac{x}{b} \Rightarrow b = \frac{x}{\tan 40°}$. Since $a + b = 100$, $\frac{x}{\tan 60°} + \frac{x}{\tan 40°} = 100$.

9. **Correct Response: A.** (SMR Code: 3.1) Constructing an angle bisector in a triangle displays a locus of points that are equidistant from the two sides of the triangle that form the angle. The point at which the three angle bisectors intersect is equidistant from the three sides of a triangle. Thus, bisecting one of the angles in the triangular park is an appropriate first step.

10. **Correct Response: C.** (SMR Code: 3.1) If the perimeter of a regular hexagon is 72 cm, then each edge has a length of 12 cm. A regular hexagon can be split into six equilateral triangles. The height of each triangle is $6\sqrt{3}$ cm, by the Pythagorean theorem, and the area of each triangle equals $\frac{1}{2}(12)(6\sqrt{3}) = 36\sqrt{3}$ cm². Thus the area of the hexagon is $216\sqrt{3}$ cm², or approximately 374 cm².

11. **Correct Response: D.** (SMR Code: 3.2) Use the fact that $PS = 2PT$ to write the equation $\sqrt{(x-4)^2 + (y-0)^2} = 2\sqrt{(x+2)^2 + (y-0)^2}$. Then squaring both sides gives $x^2 - 8x + 16 + y^2 = 4(x^2 + 4x + 4 + y^2)$, which simplifies to $3x^2 + 24x + 3y^2 = 0$. This is the equation of a circle. To find the center, complete the square: $3(x^2 + 8x) + 3y^2 = 0 \Rightarrow 3(x^2 + 8x + 16) + 3y^2 = 48 \Rightarrow (x + 4)^2 + y^2 = 16$. The circle's center is at $(-4, 0)$.

12. **Correct Response: A.** (SMR Code: 3.3) Since line $k$ is perpendicular to plane $\alpha$, it is parallel to plane $\beta$. Because line $k$ is skew to $m$, it does not intersect $m$, and it is not contained in $\beta$. Therefore, A is the correct response.

13. **Correct Response: A.** (SMR Code: 3.3) If a line is perpendicular to two distinct lines in a plane, then it is perpendicular to the plane. This means that it is perpendicular to any line in the plane that passes through its foot. Since the intersection of $r$ and $t$ is its foot, the statement given in response A is true.

14. **Correct Response: A.** (SMR Code: 3.3) The surface area of a cylinder is $2\pi r^2 + 2\pi rh$. If the height is 10 in. and the surface area is 245 in.², then $2\pi r^2 + 20\pi r = 245$. This equation approximately equals $r^2 + 10r - 39 = 0$, which factors into $(r + 13)(r - 3) = 0$. Solving for $r$ gives $r = -13$ or 3, so the radius is approximately 3 cm. The volume of a cylinder is $\pi r^2 h$, so this cylinder has a volume of approximately $90\pi$ in.³, or about 283 in.³.

15. **Correct Response: D.** (SMR Code: 3.3) Each cut through the center of a sphere creates two circles on the surface of the resulting two hemispheres. Thus, for one cut, the total surface area is the area of the sphere and the area of the two circles formed, which is $4\pi r^2 + 2\pi r^2$. For $n$ cuts, the total surface area is equal to $4\pi r^2 + 2n\pi r^2$.

16. **Correct Response: A.** (SMR Code: 3.3) The shaded section is a trapezoidal prism, so its volume is the product of the area of the trapezoid and the height of the prism (200 cm).
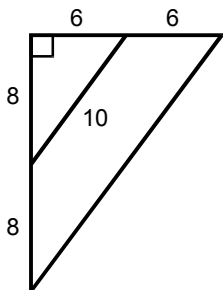


The triangle in the top left corner has legs of 8 cm and 6 cm, so its area is $\left(\frac{1}{2}\right)(8)(6) = 24$ cm².

The area of the larger triangle is $\left(\frac{1}{2}\right)(16)(12) = 96$ cm².

The trapezoid's area is the difference between the area of the large triangle (96 cm²) and the area of the small triangle (24 cm²), so the area of the trapezoid is 72 cm². Multiplying the area of the trapezoid by the height of the prism gives a volume of 14,400 cm³.

17. **Correct Response: B.** (SMR Code: 3.4) If the composition of these two transformations were commutative, it would not matter which reflection was done first. However, reflecting $\triangle ABC$ first across line $m$ and then across line $n$ would not result in $\triangle A'B'C'$, so the composition of these two transformations is not commutative.

18. **Correct Response: C.** (SMR Code: 3.4) The matrix transformation given by $T(x, y) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ is the

same as $T:(x + y, y)$. Apply $T$ to the vertices of the square in Diagram 1, as follows: $T:(0, 0) \Rightarrow (0, 0)$,

$T:(1, 0) \Rightarrow (1, 0)$, $T:(1, 1) \Rightarrow (2, 1)$, and $T:(0, 1) \Rightarrow (1, 1)$. The resulting points are the vertices of the

image in Diagram 2.

19. **Correct Response: B.** (SMR Code: 3.4) If two lines, $m_1$ and $m_2$, intersect at a point $O$, and the angle of rotation of $m_1$ to $m_2$ is $\theta$, then the result of reflecting a point over $m_1$ and its image over $m_2$ is the same as rotating the point $2\theta$ about $O$. Hence, the reflection of the points of plane figure $ABC$ over two perpendicular lines is the same as a 180° rotation.

20. **Correct Response: B.** (SMR Code: 3.4) Since $T$ is a linear transformation, one need only check how

$T$ transforms the area of a square with vertex coordinates (0, 0), (0, 1), (1, 1), and (1, 0). This square has

area 1. The image of these points is (0, 0), $\left(0, \frac{1}{2}\right)$, $\left(3, \frac{1}{2}\right)$, and (3, 0). This is a rectangle of area $\frac{3}{2}$. Hence,

$T$ maps each unit square to a rectangle of area $\frac{3}{2}$. It follows that if $A$ is the area of the original hexagon,

$\frac{3}{2}A$ is the area of the transformed hexagon, and the ratio of the original hexagon to the transformed

hexagon is $\frac{2}{3}$.

## Probability and Statistics

21. **Correct Response: A.** (SMR Code: 4.1) Each question has 5 possible answers, so the number of ways in which a student could answer one question is 5. Since there are 50 questions, the number of possible sets of answers is $5 \cdot 5 \cdot 5 \cdot \ldots \cdot 5$ (50 times), or $5^{50}$.

22. **Correct Response: D.** (SMR Code: 4.1) The event "at least one defective laptop chosen" is the complement of the event "no defective laptops are chosen." The probability, $P$, that no defective laptops are chosen in a sample of 3 equals the number of samples of 3 from the set of laptops that are not defective divided by the number of samples of 3 from the set of all of the laptops. The complement of $P$ is the probability that at least one laptop in the sample of 3 is defective and is given by

$$1 - P = 1 - \frac{C_3^{16}}{C_3^{22}} = \frac{\frac{16!}{13!3!}}{\frac{22!}{19!3!}} = 0.636.$$

23. **Correct Response: C.** (SMR Code: 4.1) If the mean of a normally distributed data set is 12 fl. oz. and the standard deviation is 0.05 fl. oz., then approximately 67% of the points in the data set have values within 0.05 fl. oz. of the mean and approximately 95% of the data points are within 0.1 fl. oz. (i.e., two standard deviations) of the mean, that is, between 11.9 fl. oz. and 12.1 fl. oz. This means that 2.5% of the data points are below 11.9 fl. oz., and 2.5% are above 12.1 fl. oz.

24. **Correct Response: D.** (SMR Code: 4.1) To find the probability, first standardize the data points using $z = \frac{x - \mu}{\sigma}$. So $z(40) = \frac{40 - 40}{1.5} = 0$, $z(39) = \frac{39 - 40}{1.5} = -\frac{2}{3}$, and $z(42) = \frac{42 - 40}{1.5} = \frac{4}{3}$. Hence, the probability that $39 \le x \le 42$ is equal to the shaded area of a normal curve that begins at $\frac{2}{3}$ of a standard deviation to the left of zero (the mean) and ends at $\frac{4}{3}$ of a standard deviation to the right of the mean.

25. **Correct Response: D.** (SMR Code: 4.2) If each measurement is exactly 1.5 ounces heavier than its true weight, then the histogram depicting the spread of measured weights will have the same shape as a histogram representing the true data, but the graph will be shifted 1.5 units (ounces) to the right. Hence, the standard deviation and variance (which measure the spread of data) will be unchanged, as will the range (difference between lowest and highest value). The quartiles, however, will change, since the values are all shifted up 1.5 units.

26. **Correct Response: B.** (SMR Code: 4.2) To find the standard deviation, $\sqrt{\frac{1}{n} \Sigma(x - \mu)^2}$, of the set of data $\{4, 9, x\}$, note that the mean, $\mu$, of the set of data is $\frac{13 + x}{3}$. The deviations of each datum from $\mu$, $(x - \mu)$, are $4 - \frac{13 + x}{3} = \frac{-x - 1}{3}$, $9 - \frac{13 + x}{3} = \frac{14 - x}{3}$, and $x - \frac{13 + x}{3} = \frac{2x - 13}{3}$. The sum of the squared deviations, $\Sigma(x - \mu)^2$, is $\frac{2x^2 - 26x + 122}{3}$ and $\frac{1}{n} = \frac{1}{3}$. Finally, the standard deviation $\sqrt{\frac{1}{3} \cdot \frac{2x^2 - 26x + 122}{3}}$ $= \sqrt{\frac{14}{3}}$. By the quadratic formula, $x = 5$ or $x = 8$. Of the available responses, B = 8 is the correct response.

27. **Correct Response: B.** (SMR Code: 4.2) The correlation coefficient is a number between $-1$ and $1$, inclusive. Therefore, the only possible values for the correlation coefficient are $-0.8$ and $0.7$. Since the slope of the regression line for the data set is $-2.7$, the most reasonable value for the correlation coefficient is $-0.8$.

# Examples of Strong Responses to Sample Constructed-Response Questions for CSET: Mathematics Subtest II

## Geometry

**Question #28  (Score Point 4 Response)**
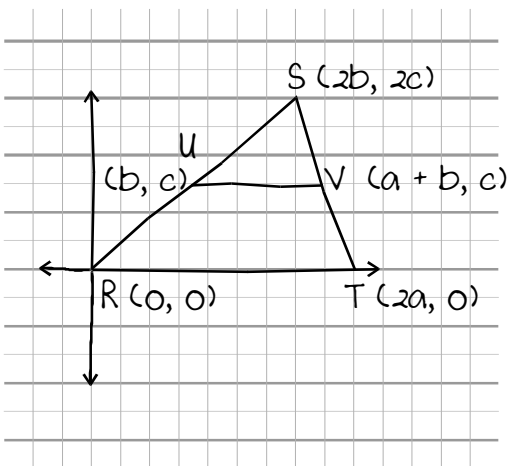
Let the coordinates of R = (0, 0), S = (2b, 2c), and T = (2a, 0).

Since U is the midpoint of RS, its coordinates are $\left(\dfrac{0 + 2b}{2}, \dfrac{0 + 2c}{2}\right)$ = (b, c).

Since V is the midpoint of TS, its coordinates are $\left(\dfrac{2a + 2b}{2}, \dfrac{0 + 2c}{2}\right)$ = (a + b, c).

So to show $\overline{UV} \parallel \overline{RT}$ means showing their slopes are the same:

Slope is $\dfrac{\Delta y}{\Delta x}$.

So, slope of $\overline{UV}$ = $\dfrac{c - c}{(a + b) - b}$ = $\dfrac{0}{a}$ = 0

slope of $\overline{RT}$ = $\dfrac{0 - 0}{2a - 0}$ = $\dfrac{0}{2a}$ = 0

Since the slopes of $\overline{UV}$ and $\overline{RT}$ are the same, the segments are parallel.

Show the length of $\overline{UV}$ = $\dfrac{1}{2}$ length of $\overline{RT}$:

The length of $\overline{UV}$ = a + b − b = a (since $\overline{UV}$ is horizontal).

The length of $\overline{RT}$ is 2a  ($\overline{RT}$ horizontal).

Therefore the length of $\overline{UV}$ = $\dfrac{1}{2}$ length of $\overline{RT}$.

**Question #29  (Score Point 4 Response)**

Let $(x, y)$ be the point on the curve.

The distance from $(x, y)$ to $(0, 0)$ is found by using the distance formula

$$d_1 = \sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{x^2 + y^2}$$

The distance from $(x, y)$ to $(0, 4)$ is $d_2 = \sqrt{(x - 0)^2 + (y - 4)^2} = \sqrt{x^2 + (y - 4)^2}$

We know $d_1 + d_2 = 12$, so $\sqrt{x^2 + y^2} + \sqrt{x^2 + (y - 4)^2} = 12$.

To square a radical equation, put the radicals on different sides of the equation and square both sides:

$$\left(\sqrt{x^2 + (y - 4)^2}\right)^2 = \left(12 - \sqrt{x^2 + y^2}\right)^2$$

$$x^2 + (y - 4)^2 = 144 - 24\sqrt{x^2 + y^2} + x^2 + y^2$$

Isolate the radical on one side again:

$$x^2 + (y - 4)^2 - 144 - x^2 - y^2 = -24\sqrt{x^2 + y^2}$$

$$y^2 - 8y + 16 - 144 - y^2 = -24\sqrt{x^2 + y^2}$$

$$-8y - 128 = -24\sqrt{x^2 + y^2}$$

$$y + 16 = 3\sqrt{x^2 + y^2}$$

Square both sides:

$$y^2 + 32y + 256 = 9(x^2 + y^2)$$

$$y^2 + 32y + 256 = 9x^2 + 9y^2$$

$$-8y^2 + 32y - 9x^2 = -256$$

**Question #29  (Score Point 4 Response)** *continued*

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Complete the square for $-8y^2 + 32y$:

$$-8(y^2 - 4y) - 9x^2 = -256$$

$$-8(y^2 - 4y + 4) - 9x^2 = -256 - 32$$

$$-8(y - 2)^2 - 9x^2 = -288$$

$$\frac{(y - 2)^2}{36} + \frac{x^2}{32} = 1$$

This is the equation of an ellipse centered at $(0, 2)$.

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## Probability and Statistics

**Question #30  (Score Point 4 Response)**

- Null Hypothesis: The current distribution is the same as that in 1999.

  Alternative Hypothesis: The current distribution is not the same as that in 1999.

- Based on the 1999 distribution, one expects:

  69.1% of 500 = 345.5  to be owner-occupied
  24.6% of 500 = 123  to be renter-occupied
  6.3% of 500 = 31.5  to be vacant

- $X^2$ – statistic = Sum of $\dfrac{(\text{Observed Frequency} - \text{Expected Frequency})^2}{\text{Expected Frequency}}$

|  | Observed Frequency | Expected Frequency | $\dfrac{(\text{Obs} - \text{Exp})^2}{\text{Exp}}$ |
|---|---|---|---|
| Owner | 375 | 345.5 | $\dfrac{(375 - 345.5)^2}{345.5} = 2.5188$ |
| Renter | 100 | 123 | $\dfrac{(100 - 123)^2}{123} = 4.3008$ |
| Vacant | 25 | 31.5 | $\dfrac{(25 - 31.5)^2}{31.5} = 1.3413$ |
| Total | 500 | 500 |  |
|  |  |  | $X^2 = \text{Sum of } \dfrac{(\text{Obs Freq} - \text{Exp Freq})^2}{\text{Exp Freq}} = 8.1609$ |

- For a $X^2$ – statistic of 8.1609, the probability lies between 0.01 and 0.025, i.e., the P-value is between 1% and 2.5%.

  Therefore, at the 5% significance level, the null hypothesis is rejected and one concludes that the data suggest that the distribution of current housing units in U.S. suburban areas differs from the 1999 distribution.

# Scoring Information for CSET: Mathematics Subtest II

Responses to the multiple-choice questions are scored electronically.  Scores are based on the number of questions answered correctly.  There is no penalty for guessing.

There are three constructed-response questions in Subtest II of CSET: Mathematics.  Each of these constructed-response questions is designed so that a response can be completed within a short amount of time—approximately 10–15 minutes.  Responses to constructed-response questions are scored by qualified California educators using focused holistic scoring.  Scorers will judge the overall effectiveness of your responses while focusing on the performance characteristics that have been identified as important for this subtest (see below).  Each response will be assigned a score based on an approved scoring scale (see page 30).

Your performance on the subtest will be evaluated against a standard determined by the Commission on Teacher Credentialing based on professional judgments and recommendations of California educators.

## Performance Characteristics for CSET: Mathematics Subtest II

The following performance characteristics will guide the scoring of responses to the constructed-response questions on CSET: Mathematics Subtest II.

| PURPOSE | The extent to which the response addresses the constructed-response assignment's charge in relation to relevant CSET subject matter requirements. |
| --- | --- |
| SUBJECT MATTER KNOWLEDGE | The application of accurate subject matter knowledge as described in the relevant CSET subject matter requirements. |
| SUPPORT | The appropriateness and quality of the supporting evidence in relation to relevant CSET subject matter requirements. |
| DEPTH AND BREADTH OF UNDERSTANDING | The degree to which the response demonstrates understanding of the relevant CSET subject matter requirements. |

## Scoring Scale for CSET: Mathematics Subtest II

Scores will be assigned to each response to the constructed-response questions on CSET: Mathematics Subtest II according to the following scoring scale.

| SCORE POINT | SCORE POINT DESCRIPTION |
|---|---|
| **4** | **The "4" response reflects a thorough command of the relevant knowledge and skills as defined in the subject matter requirements for CSET: Mathematics.**<br>• The purpose of the assignment is fully achieved.<br>• There is a substantial and accurate application of relevant subject matter knowledge.<br>• The supporting evidence is sound; there are high-quality, relevant examples.<br>• The response reflects a comprehensive understanding of the assignment. |
| **3** | **The "3" response reflects a general command of the relevant knowledge and skills as defined in the subject matter requirements for CSET: Mathematics.**<br>• The purpose of the assignment is largely achieved.<br>• There is a largely accurate application of relevant subject matter knowledge.<br>• The supporting evidence is adequate; there are some acceptable, relevant examples.<br>• The response reflects an adequate understanding of the assignment. |
| **2** | **The "2" response reflects a limited command of the relevant knowledge and skills as defined in the subject matter requirements for CSET: Mathematics.**<br>• The purpose of the assignment is partially achieved.<br>• There is limited accurate application of relevant subject matter knowledge.<br>• The supporting evidence is limited; there are few relevant examples.<br>• The response reflects a limited understanding of the assignment. |
| **1** | **The "1" response reflects little or no command of the relevant knowledge and skills as defined in the subject matter requirements for CSET: Mathematics.**<br>• The purpose of the assignment is not achieved.<br>• There is little or no accurate application of relevant subject matter knowledge.<br>• The supporting evidence is weak; there are no or few relevant examples.<br>• The response reflects little or no understanding of the assignment. |
| **U** | **The "U" (Unscorable) is assigned to a response that is unrelated to the assignment, illegible, primarily in a language other than English, or does not contain a sufficient amount of original work to score.** |
| **B** | **The "B" (Blank) is assigned to a response that is blank.** |