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Subject
Examinations for
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TEST GUIDE

MATHEMATICS
SUBTEST III

Sample Questions and Responses and Scoring Information

CS-TG-QR213X-01

Sample Test Questions for CSET: Mathematics Subtest III

Below is a set of multiple-choice questions and constructed-response questions that are similar to the questions you will see on Subtest III of CSET: Mathematics. Please note that, as on the actual test form, approximately one third of the multiple-choice questions in this test guide are more complex questions that require 2–3 minutes each to complete. You are encouraged to respond to the questions without looking at the responses provided in the next section. Record your responses on a sheet of paper and compare them with the provided responses.

Note: The use of calculators is not allowed for CSET: Mathematics Subtest III.

Note: In CSET: Mathematics subtests, log *x* represents the base-10 logarithm of *x*.

1. What are the solutions to $\tan\left(x + \frac{\pi}{6}\right) = \sqrt{3}$ where $0 \le x \le 2\pi$?

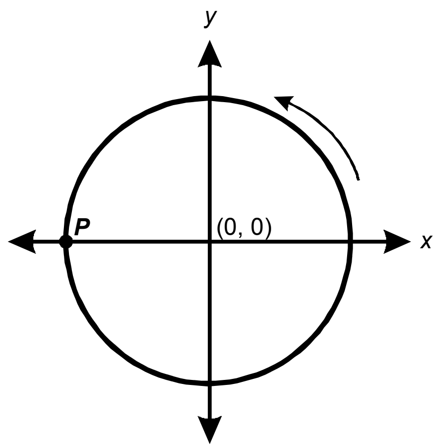
   A. $\frac{11\pi}{6}, \frac{5\pi}{6}$

   B. $\frac{5\pi}{3}, \frac{2\pi}{3}$

   C. $\frac{\pi}{3}, \frac{4\pi}{3}$

   D. $\frac{\pi}{6}, \frac{7\pi}{6}$

2. **Use the diagram below to answer the question that follows.**



Point P is on the edge of a wheel of radius of 1 that rotates counterclockwise around the origin at a speed of 15 revolutions per second. At $t = 0$, P has coordinates $(-1, 0)$. Which of the following functions could be used to describe the x-coordinate of P as a function of time, t, measured in seconds?

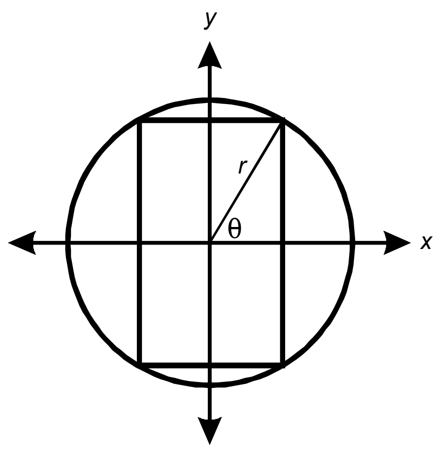
A. $x(t) = \sin(30\pi t) - 1$

B. $x(t) = -\sin\left(\frac{2\pi t}{15}\right) - 1$

C. $x(t) = -\cos(30\pi t)$

D. $x(t) = -\cos\left(\frac{2\pi t}{15}\right)$

3. **Use the diagram below to answer the question that follows.**



The diagram shows a rectangle inscribed in a circle of radius r. What is the area of the rectangle as a function of the angle θ?

A. $4r \tan \theta$

B. $r^2 \left|\cos \theta \sin \theta\right|$

C. $4r^2 \tan^2\theta$

D. $4r^2 \left|\cos \theta \sin \theta\right|$

4. Which of the following expressions is equal to $\sin\left(\arccos \frac{x}{3}\right)$?

A. $\frac{\sqrt{9 - x^2}}{3}$

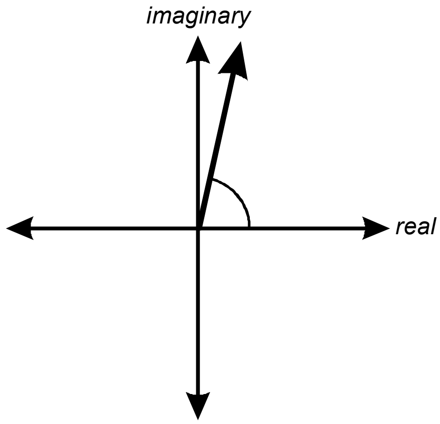
B. $\frac{\pm\sqrt{9 - x^2}}{3}$

C. $\frac{\sqrt{9 + x^2}}{x}$

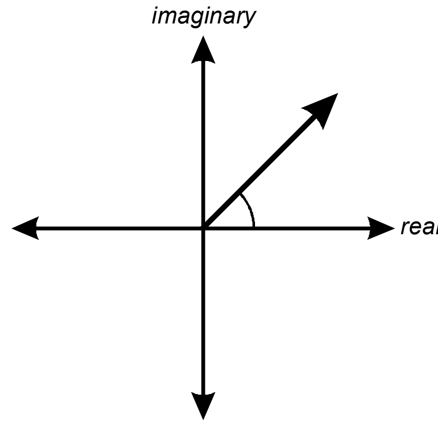
D. $\frac{\sqrt{9 + x^2}}{|x|}$

5. Given that $z = \cos 75° + i \sin 75°$, which of the following best represents z^3 in vector form?

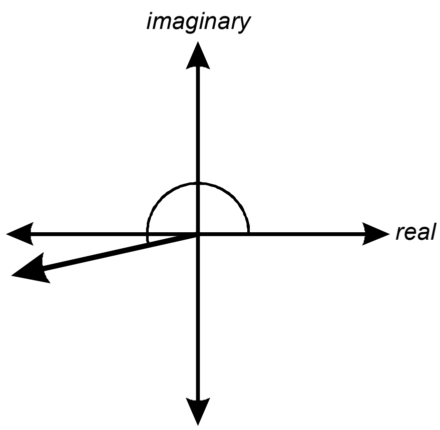
A.



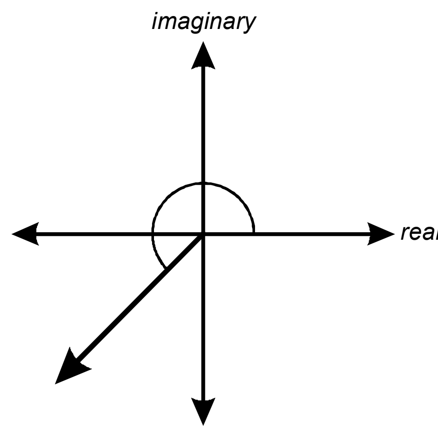
B.



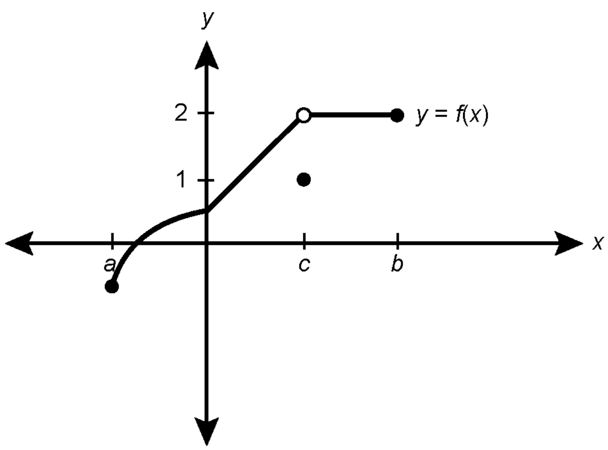
C.



D.



6. **Use the graph below to answer the question that follows.**



Let $f(x)$ be the function whose graph is shown above. Which of the following statements best describes why $f(x)$ is discontinuous at $x = c$?

A. $f(c)$ does not exist

B. $\lim_{x \to c} f(x) \neq f(c)$

C. $\lim_{x \to c} f(x)$ does not exist

D. $f'(c) \neq f'(c + \Delta x)$

7. Let $f(x)$ and $g(x)$ be functions such that $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$ exist. If $\lim_{x \to c} [f(x) + g(x)] = 6$ and $\lim_{x \to c} [2f(x) - g(x)] = 1$, what is $\lim_{x \to c} f(x)$?

   A. $\frac{7}{2}$

   B. $\frac{7}{3}$

   C. $\frac{5}{2}$

   D. $\frac{5}{3}$

8. For which of the following values of k does $\lim_{x \to k} \dfrac{\sin x + \sin x \cos^2 x}{\sin x \cos x}$ equal -2?

   A. $\frac{\pi}{2}$

   B. π

   C. $\frac{3\pi}{2}$

   D. 2π

9. Given the function $g(x) = \dfrac{2x^2 - 32}{3x^2 - 9x - 12}$ for $x \neq 4$, define $g(4)$ so that $g(x)$ is continuous at $x = 4$.

   A. $g(4) = 0$

   B. $g(4) = \frac{2}{3}$

   C. $g(4) = \frac{16}{15}$

   D. $g(4) = \frac{8}{3}$

10. If $f(x)$ is continuous on the interval $[a, b]$, and N is any number strictly between $f(a)$ and $f(b)$, which of the following must be true?

    A. The inverse of $f(x)$ is defined on the interval $[f(a), f(b)]$.

    B. There exists a number c in (a, b) such that $f(c) = N$.

    C. $f(x)$ is differentiable on the interval (a, b).

    D. There exists a number c in (a, b) such that $f'(c) = 0$.

11. An economist needs to approximate the function $f(x) = \frac{1}{x}$ by a line tangent to $f(x)$ at $x = -1$. Which of the following lines should be used?

    A. $y = -x - 1$

    B. $y = -x - 2$

    C. $y = -2x - 3$

    D. $y = -3x - 4$

12. Determine $\lim_{x \to 0} \frac{\sin(x) - x}{x^3}$.

    A. $-\frac{1}{6}$

    B. 0

    C. $\frac{1}{6}$

    D. ∞

13. Given $f(x) = x^3$, for what value of x does the instantaneous rate of change of $f(x)$ equal the average rate of change of $f(x)$ on the interval $1 \leq x \leq 4$?

    A. $x = \sqrt[3]{\frac{14}{3}}$

    B. $x = \sqrt{\frac{17}{2}}$

    C. $x = \sqrt[3]{21}$

    D. $x = \sqrt{7}$

14. A curve in the *x*-*y* plane is defined by $x^2 + xy + y^2 = 1$. For what values of x is the line tangent to the curve horizontal?

    A. $x = -2$ and $x = 1$

    B. $x = \frac{-\sqrt{3}}{3}$ and $x = \frac{\sqrt{3}}{3}$

    C. $x = \frac{-2\sqrt{3}}{3}$ and $x = \frac{2\sqrt{3}}{3}$

    D. $x = \frac{1 + \sqrt{3}}{4}$ and $x = \frac{1 - \sqrt{3}}{4}$

15. The rate at which the mass of a radioactive isotope changes with respect to time is directly proportional to the mass of the isotope, where k is the constant of proportionality. If 500 grams of the isotope are present at time $t = 0$ and 100 grams are left at time $t = 20$, what is the value of k?

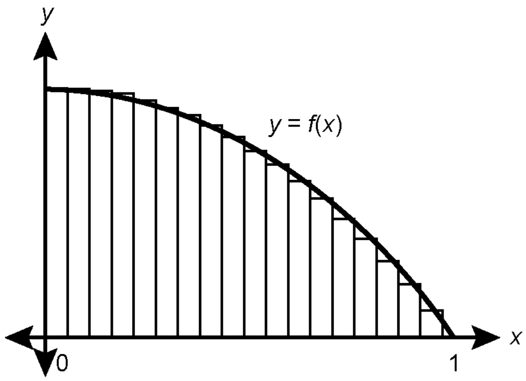
    A. $\frac{1}{20} \ln\left(\frac{1}{5}\right)$

    B. $\frac{1}{5} \ln\left(\frac{1}{20}\right)$

    C. $\frac{1}{5} \ln\left(\frac{1}{100}\right)$

    D. $\frac{1}{100} \ln\left(\frac{1}{5}\right)$

16. **Use the graph below to answer the question that follows.**



The height and width of each of the above n rectangles

is given by $f(x_i)$ and $\frac{1}{n}$, respectively, where $1 \le i \le n$.

If the sum of the areas of the n rectangles is given by

$\sum_{i=1}^{n} \frac{1}{n} f(x_i) = \frac{108n^2 - 81n - 27}{6n^2}$, then by the use of Riemann

sums $\int_0^1 f(x)\,dx$ is equal to which of the following?

A. 4.5

B. 13.5

C. 18

D. 108

17. Evaluate $\int_0^1 x(x + 1)^4\, dx$.

A. $\frac{1}{5}$

B. $\frac{16}{5}$

C. $\frac{28}{10}$

D. $\frac{43}{10}$

18. The acceleration of a particle traveling in one dimension along the x-axis at time t is given by $\frac{d^2x}{dt^2} = 6t - 2$. At time $t = 1$, the particle's position is at $x = 10$ and its velocity is 0. What is the position of the particle when $t = 4$?

A. 22

B. 39

C. 44

D. 55

19. Which of the following represents the area under the curve of the function $h(x) = \frac{4}{3x + 2}$ on the interval [0, 2]?

A. $\frac{5}{16}$

B. $\frac{15}{16}$

C. 4 ln 4

D. $\frac{4}{3}$ ln 4

20. What is the volume of the solid formed by revolving the region bounded by $y = \sqrt{x}$ and $y = \frac{x}{2}$ about the x-axis?

A. $\frac{8\pi}{3}$

B. $\frac{4\pi}{3}$

C. $\frac{8\pi}{15}$

D. $\frac{16\pi}{9}$

21. What is the sum of the series given by $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}e^{n-1}}{3^n}$?

A. $\frac{1}{3+e}$

B. $\frac{-e}{3}$

C. $\frac{1}{3-e}$

D. $\frac{e}{3}$

22. For what values of x does the infinite series $1 - (x - 2) + (x - 2)^2 - (x - 2)^3 + \ldots$ converge?

A. $x > 1$

B. $x < 3$

C. $1 < x < 3$

D. $x < 1$ or $x > 3$

23. For what value(s) of x does the series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n \cdot 4^n}$ converge?

A. $x = -1$

B. $x < 7$

C. $-1 \leq x < 7$

D. $-1 < x < 7$

24. What is the Taylor series representation for $f(x) = 10^x$ generated at $x = 0$?

A. $\sum_{n=0}^{\infty} \frac{1}{n!} (x)^n$

B. $\sum_{n=0}^{\infty} \ln 10^n \, (x)^n$

C. $\sum_{n=0}^{\infty} \frac{(\ln 10)^n}{n!} (x)^n$

D. $\sum_{n=1}^{\infty} \frac{(\log_{10} n)}{n!} (x)^n$

Constructed-Response Assignments

Read each assignment carefully before you begin your response. Think about how you will organize your response. An erasable notebooklet will be provided at the test center for you to make notes, write an outline, or otherwise prepare your response. For the examination, your final responses to each constructed-response assignment must be either:

1) typed into the on-screen response box,
2) written on a response sheet and scanned using the scanner provided at your workstation, or
3) provided using both the on-screen response box (for typed text) and a response sheet (for calculations or drawings) that you will scan using the scanner provided at your workstation.

25. **Complete the exercise that follows.**

Graph the system of equations below on the same coordinate system and then use analytic techniques to find the coordinates of the points of intersection of the graphs for $-2\pi \leq x \leq 2\pi$.

$y = \sin^2 x$
$y = \cos(x) + 1$

26. **Use the fundamental theorem of calculus stated below to complete the exercise that follows.**

> Let f be continuous on a closed interval $[a, b]$ and let x be any point in $[a, b]$. If F is defined by
>
> $$F(x) = \int_a^x f(t)\,dt,$$
>
> then $F'(x) = f(x)$ at each point x in the interval $[a, b]$.

Using the formal definition of the derivative, prove the above theorem.

Annotated Responses to Sample Multiple-Choice Questions for CSET: Mathematics Subtest III

Calculus

1. **Correct Response: D.** (SMR Code: 5.1) $\tan\left(x + \frac{\pi}{6}\right) = \sqrt{3}, \Rightarrow \left(x + \frac{\pi}{6}\right) = \arctan\sqrt{3} \Rightarrow \left(x + \frac{\pi}{6}\right) = \frac{\pi}{3}$ or $\frac{4\pi}{3}$, for $0 \le x \le 2\pi$, since $\tan\frac{\pi}{3} = \sqrt{3}$ and $\tan\frac{4\pi}{3} = \sqrt{3}$. Hence, either $x + \frac{\pi}{6} = \frac{\pi}{3}$ or $x + \frac{\pi}{6} = \frac{4\pi}{3} \Rightarrow x = \frac{\pi}{6}$ or $\frac{7\pi}{6}$.

2. **Correct Response: C.** (SMR Code: 5.1) The x-coordinate of point P on the unit circle is the cosine of the angle measured from (1, 0) to point P, θ, $\Rightarrow x = \cos\theta$. Since the wheel turns 15 revolutions per second and each revolution is equivalent to a rotation of 2π radians, the change in the angle of rotation is 30π radians per second, i.e., $\theta = 30\pi t \Rightarrow x(t) = \cos(30\pi t)$. The initial x-coordinate is -1, so when $t = 0$, $x(t)$ should equal -1. Hence the equation is $x(t) = -\cos(30\pi t)$.

3. **Correct Response: D.** (SMR Code: 5.1) To find the area of the inscribed rectangle, find the area of the rectangle in Quadrant I and multiply its area by 4. The length of the horizontal side of this rectangle is $|r\cos\theta|$, and its vertical side is $|r\sin\theta|$, so the area of the rectangle is $|r\cos\theta| \bullet |r\sin\theta| = r^2|\sin\theta\cos\theta|$. Thus the large rectangle has an area of $4r^2|\sin\theta\cos\theta|$.

4. **Correct Response: A.** (SMR Code: 5.1) The $\arccos\left(\frac{x}{3}\right)$ is the inverse cosine of an angle θ in a right triangle, so $\arccos\left(\frac{x}{3}\right) = \theta$. Therefore, the $\cos\theta = \frac{x}{3}$ and the side adjacent to the angle has length x, while the hypotenuse of the triangle has length 3. By the Pythagorean theorem, the length of the third side of the triangle is $\sqrt{9 - x^2}$. Thus, the sine of $\theta = \sin\left(\arccos\left(\frac{x}{3}\right)\right)$ is the ratio of the length of the opposite side over the hypotenuse or $\sin\theta = \frac{\sqrt{9 - x^2}}{3}$.

5. **Correct Response: D.** (SMR Code: 5.1) By DeMoivre's Theorem,

   $z^3 = 1^3\,[\cos(3 \bullet 75°) + i\sin(3 \bullet 75°)] = \cos 225° + i\sin 225°.$

   Since the argument of z is 225°, response choice D is the correct response.

6. **Correct Response: B.** (SMR Code: 5.2) Note from the graph of $y = f(x)$ that $\lim\limits_{x \to c^+} f(x) = 2$ and $\lim\limits_{x \to c^-} f(x) = 2$; thus $\lim\limits_{x \to c} f(x) = 2$. Also note from the graph of $y = f(x)$ that $f(c) = 1$. Since $\lim\limits_{x \to c} f(x) \neq f(c)$, $f(x)$ is discontinuous at $x = c$.

7. **Correct Response: B.** (SMR Code: 5.2) From the properties of limits, $\lim\limits_{x \to c}[f(x) + g(x)] = 6 \Rightarrow \lim\limits_{x \to c} f(x) + \lim\limits_{x \to c} g(x) = 6$ and $\lim\limits_{x \to c} g(x) = 6 - \lim\limits_{x \to c} f(x)$. Likewise, $\lim\limits_{x \to c}[2f(x) - g(x)] = 1 \Rightarrow 2\lim\limits_{x \to c} f(x) - \lim\limits_{x \to c} g(x) = 1$ and $\lim\limits_{x \to c} g(x) = 2\lim\limits_{x \to c} f(x) - 1$. Since $\lim\limits_{x \to c} g(x) = \lim\limits_{x \to c} g(x)$, then $2\lim\limits_{x \to c} f(x) - 1 = 6 - \lim\limits_{x \to c} f(x) \Rightarrow 2\lim\limits_{x \to c} f(x) + \lim\limits_{x \to c} f(x) = 6 + 1 \Rightarrow 3\lim\limits_{x \to c} f(x) = 7 \Rightarrow \lim\limits_{x \to c} f(x) = \frac{7}{3}$.

8. **Correct Response: B.** (SMR Code: 5.2) Begin by simplifying the expression:

$\lim\limits_{x \to k} \dfrac{\sin x + \sin x \cos^2 x}{\sin x \cos x} = \lim\limits_{x \to k} \dfrac{\sin x\,(1 + \cos^2 x)}{\sin x \cos x} = \lim\limits_{x \to k} \dfrac{1 + \cos^2 x}{\cos x} = \dfrac{1 + \cos^2 k}{\cos k}$. To find the value of k that makes this limit equal to -2, solve: $\dfrac{1 + \cos^2 k}{\cos k} = -2 \Rightarrow 1 + \cos^2 k = -2 \cos k \Rightarrow \cos^2 k + 2 \cos k + 1 = 0$ $\Rightarrow (\cos k + 1)(\cos k + 1) = 0 \Rightarrow \cos k = -1$. The angle with a cosine of -1 is π radians, so $k = \pi$.

9. **Correct Response: C.** (SMR Code: 5.2) The function $g(x)$ is continuous at $x = 4$ if and only if $\lim\limits_{x \to 4} g(x) = g(4)$. Since $g(4)$ is undefined, define $g(4) = \lim\limits_{x \to 4} \dfrac{2x^2 - 32}{3x^2 - 9x - 12} = \lim\limits_{x \to 4} \dfrac{2(x + 4)(x - 4)}{3(x + 1)(x - 4)} = \dfrac{2}{3} \lim\limits_{x \to 4} \dfrac{(x + 4)}{(x + 1)} = \dfrac{2(4 + 4)}{3(4 + 1)} = \dfrac{16}{15}$.

10. **Correct Response: B.** (SMR Code: 5.2) The intermediate value theorem states that if a function is continuous on $[a, b]$ and if $f(a) \neq f(b)$, then somewhere in the interval (a, b) the function must take on each value between $f(a)$ and $f(b)$. So if $f(a) < N < f(b)$, the function must equal N at some point in (a, b).

11. **Correct Response: B.** (SMR Code: 5.3) If $f(x) = \dfrac{1}{x}$, then $f'(x) = -\dfrac{1}{x^2}$, so the slope of the line tangent to $f(x)$ at -1 is $\dfrac{-1}{(-1)^2}$ or -1. Then the equation of the tangent line is $y = -x + b$. To find b, substitute the values $x = -1$ and $y = f(-1) = -1$ into $y = -x + b$, which gives $b = -2$. Thus, the equation of the line tangent to $f(x)$ is $y = -x - 2$.

12. **Correct Response: A.** (SMR Code: 5.3) Since substituting $x = 0$ into $\dfrac{\sin(x) - x}{x^3}$ yields $\dfrac{0}{0}$, L'Hôpital's rule can be used to calculate this limit. Consecutive applications of L'Hôpital's rule (until substituting $x = 0$ does not result in an indeterminate form) gives:

$\lim\limits_{x \to 0} \dfrac{\sin(x) - x}{x^3} = \lim\limits_{x \to 0} \dfrac{\cos(x) - 1}{3x^2} = \lim\limits_{x \to 0} \dfrac{-\sin x}{6x} = \lim\limits_{x \to 0} \dfrac{-\cos x}{6}$. Since $\cos 0 = 1$, the limit is $-\dfrac{1}{6}$.

13. **Correct Response: D.** (SMR Code: 5.3) The average rate of change is given by $\frac{\Delta f(x)}{\Delta x}$. Over the interval $1 \le x \le 4$, the average rate of change $= \frac{f(4) - f(1)}{4 - 1} \Rightarrow \frac{4^3 - 1^3}{4 - 1} = \frac{63}{3} = 21$. The instantaneous rate of change is given by the first derivative of $f(x) = x^3$, or $f'(x) = 3x^2$. The average rate of change equals the instantaneous rate of change over the given interval when $3x^2 = 21$ or $x = \pm\sqrt{7}$. Hence, response D is the correct response.

14. **Correct Response: B.** (SMR Code: 5.3) The slope of a tangent line is given by $\frac{dy}{dx}$. Horizontal lines have a slope of 0; therefore, $\frac{dy}{dx} = 0$. By implicit differentiation, $\frac{d}{dx}(x^2 + xy + y^2) = \frac{d}{dx}(1) \Rightarrow$ $\frac{d}{dx}(x^2) + \frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = \frac{d}{dx}(1) \Rightarrow 2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$. Therefore, $\frac{dy}{dx} = \frac{-y - 2x}{x + 2y}$. Setting this equal to zero results in the following: $\frac{dy}{dx} = 0 = \frac{-y - 2x}{x + 2y} \Rightarrow -y - 2x = 0 \Rightarrow y = -2x$. Substituting this value for y into $x^2 + xy + y^2 = 1$ gives $x^2 + x(-2x) + (-2x)^2 = 1$ or $x = \pm\frac{\sqrt{3}}{3}$.

15. **Correct Response: A.** (SMR Code: 5.3) This situation is modeled by the differential equation $\frac{dM}{dt} = kM$ where M is the mass and t represents time. This is a separable differential equation and can be rewritten as $\frac{dM}{M} = k\,dt$. Integrating each side of the equation gives $\ln(M) = kt + c$. Using $M = 500$ grams when $t = 0$ gives $c = \ln(500)$. Using $M = 100$ grams when $t = 20$ gives $\ln(100) = 20k + \ln(500) \Rightarrow k = \frac{1}{20}\ln\left(\frac{1}{5}\right)$.

16. **Correct Response: C.** (SMR Code: 5.4) Calculating the area under a curve from $x = 0$ to $x = 1$ can be done either by finding $\int_0^1 f(x)\,dx$ or by finding its equivalent, $\lim_{n\to\infty}\sum_{i=1}^{n} \frac{1}{n} f(x_i)$. Since $\lim_{n\to\infty}\left(\sum_{i=1}^{n} \frac{1}{n} f(x_i)\right) = \lim_{n\to\infty} \frac{108n^2 - 81n - 27}{6n^2} = \lim_{n\to\infty} \frac{108 - \frac{81}{n} - \frac{27}{n^2}}{6} = \frac{108}{6} = 18$, $\int_0^1 f(x)\,dx = 18$.

17. **Correct Response: D.** (SMR Code: 5.4) To integrate this function, use integration by parts, letting $u = x$ and $dv = (x + 1)^4\,dx$. Then $du = dx$, $v = \frac{1}{5}(x + 1)^5$, and

$$\int_0^1 x(x + 1)^4\,dx = uv - \int v\,du = \frac{1}{5}x(x + 1)^5 \Big|_0^1 - \int_0^1 \frac{1}{5}(x + 1)^5\,dx = \frac{32}{5} - \frac{1}{30}(x + 1)^6 \Big|_0^1 = \frac{43}{10}.$$

18. **Correct Response: D.** (SMR Code: 5.4) Since $x(t)$ is the position of the particle with respect to time, $v(t) = \frac{dx}{dt}$ and $a(t) = \frac{dv}{dt}$. Hence, $v(t) = \int (6t - 2)\,\mathrm{d}t = 3t^2 - 2t + C$. To find the value of C, note that $v(1) = 0$ so that $0 = 3(1)^2 - 2(1) + C \Rightarrow C = -1$. Also, $x(t) = \int (3t^2 - 2t - 1)\,dt = t^3 - t^2 - t + C_2$. To find the value of C_2, use the fact that $x(1) = 10$ so that $10 = (1)^3 - (1)^2 - 1 + C_2 \Rightarrow C_2 = 11$. The position when $t = 4$ is found by substituting 4 into $x(t) = t^3 - t^2 - t + 11$ or $x(4) = (4)^3 - (4)^2 - 4 + 11 = 55$.

19. **Correct Response: D.** (SMR Code: 5.4) Finding the area under the curve of $h(x)$ involves computing the integral $\int_0^2 \frac{4}{3x+2}\,dx$. Using substitution, where $u = 3x + 2$ and $du = 3\,dx$, $\int_0^2 \frac{4}{3x+2}\,dx = \frac{4}{3}\int_2^8 \frac{1}{u}\,du = \frac{4}{3}\ln u \big|_2^8 = \frac{4}{3}\ln 4$.

20. **Correct Response: A.** (SMR Code: 5.4) By the washer (disk) method, the volume is given by $V = \pi\int_a^b (f(x)^2 - g(x)^2)dx$, where $f(x)$ is the radius of the outer circle and $g(x)$ is the radius of the inner circle of a washer of thickness dx. Note that the radius of the outer circle is $y = \sqrt{x}$ and the radius of the inner circle is given by $y = \frac{x}{2}$. To find the values of a and b it is necessary to find the points where $\sqrt{x} = \frac{x}{2}$ or $x^2 - 4x = 0 \Rightarrow x = 0$ and $x = 4$. Hence, the volume of this region is given by $\pi\int_0^4 \left(\left(\sqrt{x}\right)^2 - \left(\frac{x}{2}\right)^2\right) dx \Rightarrow \pi\left[\frac{x^2}{2} - \frac{x^3}{12}\right]_0^4 = \frac{8\pi}{3}$.

21. **Correct Response: A.** (SMR Code: 5.5) Writing the first few terms of the series results in $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}e^{n-1}}{3^n} = \frac{e^0}{3^1} - \frac{e^1}{3^2} + \frac{e^2}{3^3} - \frac{e^3}{3^4} + \ldots$. This is a geometric series of the form $\frac{1}{3}\sum_{n=0}^{\infty}\left(-\frac{e}{3}\right)^n$, where $a = \frac{1}{3}$ and $r = \frac{-e}{3}$. The sum of an infinite geometric series is given by $\frac{a}{1-r}$. Thus, the sum of the series is found by $\frac{\frac{1}{3}}{1 - \frac{-e}{3}} = \frac{1}{3+e}$.

22. **Correct Response: C.** (SMR Code: 5.5) The series $1 - (x - 2) + (x - 2)^2 - (x - 2)^3 + \ldots$ is a geometric series of the form $a + ar + ar^2 + \ldots + ar^{k-1} + \ldots$, where $a = 1$ and $r = -(x - 2)$. A geometric series of this form converges if $|r| < 1$. Therefore, this series converges if $|-(x - 2)| < 1$, i.e., $|x - 2| < 1$, which means $-1 < x - 2 < 1$. Thus, $1 < x < 3$.

23. **Correct Response: C.** (SMR Code: 5.5) By the ratio test, if $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$, the series converges. If the limit equals one or does not exist, then the test is inconclusive. For the series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n \cdot 4^n}$, $a_n = \frac{(x-3)^n}{n \cdot 4^n}$ and $a_{n+1} = \frac{(x-3)^{n+1}}{(n+1) \cdot 4^{n+1}}$. Therefore, $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{(x-3)^{n+1}}{(n+1) \cdot 4^{n+1}} \cdot \frac{n \cdot 4^n}{(x-3)^n} \right| = \lim_{n \to \infty} \left| \frac{(x-3)}{1} \cdot \frac{n \cdot 4^n}{(n+1) \cdot 4^{n+1}} \right| = |x-3| \lim_{n \to \infty} \left| \frac{n}{4n+4} \right| = |x-3| \cdot \left| \frac{1}{4} \right|$. By the ratio test, the series converges when $\left| \frac{x-3}{4} \right| < 1$ or for $-1 < x < 7$. Note that the limit equals 1 for $x = -1$ and $x = 7$, which is inconclusive. It is therefore necessary to check these values in the given series for convergence. When $x = 7$, $(x - 3) = 4$ and the resulting harmonic series diverges. When $x = -1$, the alternating series $-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \ldots$ results. By the alternating series test, this series converges since its terms are alternating and their absolute values decrease to zero. Thus, the interval of x for which the given series converges is $-1 \leq x < 7$.

24. **Correct Response: C.** (SMR Code: 5.5) The Taylor series for $f(x - a)$ has the form $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$, where $f^{(n)}$ is the nth derivative of $f(x)$ and a is the base point around which the series is generated. In this case, $f(x) = 10^x$ and $a = 0$. Taking derivatives and evaluating them for $x = 0$ results in the values below.

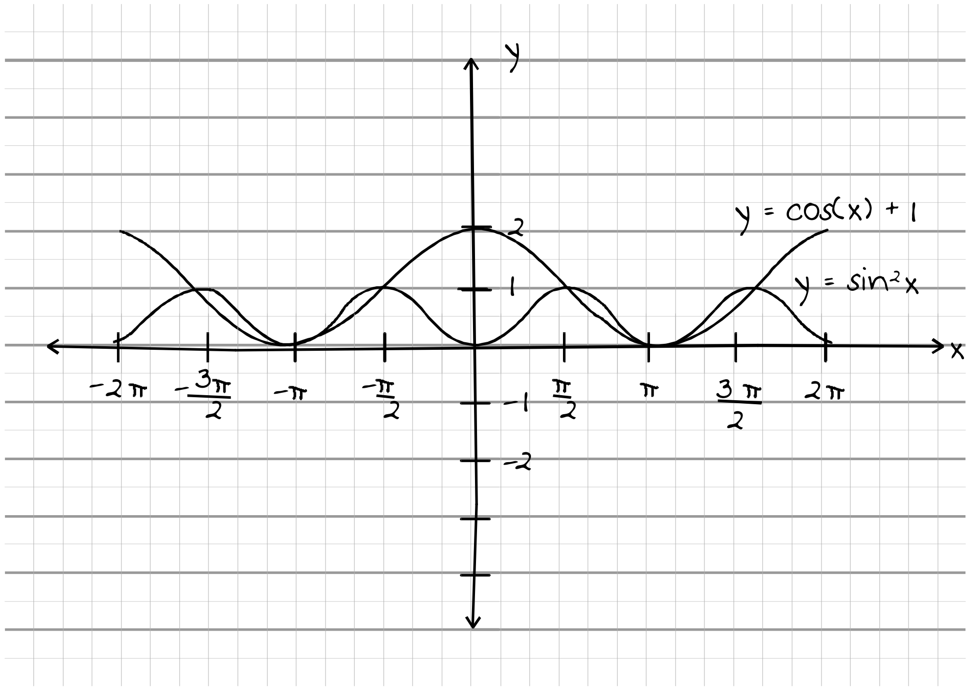
$f(x) = 10^x$	$f(0) = 1$
$f'(x) = 10^x \ln 10$	$f'(0) = \ln 10$
$f''(x) = 10^x \ln 10^2$	$f''(0) = \ln 10^2$
$f'''(x) = 10^x \ln 10^3$	$f'''(0) = \ln 10^3$

Therefore, $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = 1 + \ln 10 \cdot x + \frac{\ln 10^2}{2!} \cdot x^2 + \frac{\ln 10^3}{3!} \cdot x^3 + \ldots + \frac{\ln 10^n}{n!} \cdot x^n + \ldots$.

Examples of Strong Responses to Sample Constructed-Response Questions for CSET: Mathematics Subtest III

Calculus

Question #25 (Score Point 4 Response)



To find the points of intersection: solve the simultaneous set of equations

$$y = \sin^2 x \quad (1)$$

$$y = \cos(x) + 1 \quad (2)$$

To solve the set of equations: substitute equation (1) into (2)

$$\Rightarrow \quad \sin^2 x = \cos(x) + 1 \quad (3)$$

From the identity $\sin^2 x + \cos^2 x = 1$, $\sin^2 x = 1 - \cos^2 x \quad (4)$

Substituting equation (4) into (3) gives

$$1 - \cos^2 x = \cos(x) + 1$$

$$-\cos^2 x = \cos x$$

$$0 = \cos x + \cos^2 x$$

continued on next page

Question #25 (Score Point 4 Response) ***continued***

Factor: $0 = \cos x(1 + \cos x)$

$\Rightarrow \cos x = 0$ or $1 + \cos x = 0$, for $-2\pi < x < 2\pi$

$\cos x = -1$

$\Rightarrow x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$ or $x = -\pi, \pi$

Substitute x-values into equation (2) to get y-values:

$y = \cos\left(-\frac{3\pi}{2}\right) + 1 = 1$ $\quad$ $y = \cos\left(-\frac{\pi}{2}\right) + 1 = 1$

$y = \cos\left(\frac{\pi}{2}\right) + 1 = 1$ $\quad$ $y = \cos\left(\frac{3\pi}{2}\right) + 1 = 1$

$y = \cos(-\pi) + 1 = 0$ $\quad$ $y = \cos(\pi) + 1 = 0$

Coordinates of points of intersection are $\left(-\frac{3\pi}{2}, 1\right), \left(-\frac{\pi}{2}, 1\right), \left(\frac{\pi}{2}, 1\right), \left(\frac{3\pi}{2}, 1\right), (-\pi, 0),$ and $(\pi, 0)$.

Question #26 (Score Point 4 Response)

The derivative of F at x, $F'(x)$, is defined by: $F'(x) = \lim_{h \to 0} \dfrac{F(x+h) - F(x)}{h}$.

Using $F(x) = \int_a^x f(t)\,dt$, $F'(x) = \lim_{h \to 0} \dfrac{\int_a^{x+h} f(t)\,dt - \int_a^x f(t)\,dt}{h}$.

Since $\int_a^x f(t)\,dt = -\int_x^a f(t)\,dt$,

$$F'(x) = \lim_{h \to 0} \frac{\int_x^a f(t)\,dt + \int_a^{x+h} f(t)\,dt}{h}.$$

Notice that $\int_x^a f(t)\,dt + \int_a^{x+h} f(t)\,dt = \int_x^{x+h} f(t)\,dt$, so $F'(x) = \lim_{h \to 0} \dfrac{\int_x^{x+h} f(t)\,dt}{h}$.

The mean value theorem for integrals says that for some $x_1 \in [x, x+h]$,

$\int_x^{x+h} f(t)\,dt = f(x_1) \bullet h$. So $F'(x) = \lim_{h \to 0} \frac{1}{h}(f(x_1) \bullet h) = \lim_{h \to 0} f(x_1)$.

Since $x \le x_1 \le x + h$, x_1 approaches x as h approaches 0. Thus, $f(x_1)$ approaches $f(x)$ as h approaches 0, by the continuity of f.

Therefore, $F'(x) = \lim_{h \to 0} f(x_1) = f(x)$.

Scoring Information for CSET: Mathematics Subtest III

Responses to the multiple-choice questions are scored electronically. Scores are based on the number of questions answered correctly. There is no penalty for guessing.

There are two constructed-response questions in Subtest III of CSET: Mathematics. Each of these constructed-response questions is designed so that a response can be completed within a short amount of time—approximately 10–15 minutes. Responses to constructed-response questions are scored by qualified California educators using focused holistic scoring. Scorers will judge the overall effectiveness of your responses while focusing on the performance characteristics that have been identified as important for this subtest (see below). Each response will be assigned a score based on an approved scoring scale (see page 26).

Your performance on the subtest will be evaluated against a standard determined by the Commission on Teacher Credentialing based on professional judgments and recommendations of California educators.

Performance Characteristics for CSET: Mathematics Subtest III

The following performance characteristics will guide the scoring of responses to the constructed-response questions on CSET: Mathematics Subtest III.

PURPOSE	The extent to which the response addresses the constructed-response assignment's charge in relation to relevant CSET subject matter requirements.
SUBJECT MATTER KNOWLEDGE	The application of accurate subject matter knowledge as described in the relevant CSET subject matter requirements.
SUPPORT	The appropriateness and quality of the supporting evidence in relation to relevant CSET subject matter requirements.
DEPTH AND BREADTH OF UNDERSTANDING	The degree to which the response demonstrates understanding of the relevant CSET subject matter requirements.

Scoring Scale for CSET: Mathematics Subtest III

Scores will be assigned to each response to the constructed-response questions on CSET: Mathematics Subtest III according to the following scoring scale.

SCORE POINT	SCORE POINT DESCRIPTION
4	**The "4" response reflects a thorough command of the relevant knowledge and skills as defined in the subject matter requirements for CSET: Mathematics.** • The purpose of the assignment is fully achieved. • There is a substantial and accurate application of relevant subject matter knowledge. • The supporting evidence is sound; there are high-quality, relevant examples. • The response reflects a comprehensive understanding of the assignment.
3	**The "3" response reflects a general command of the relevant knowledge and skills as defined in the subject matter requirements for CSET: Mathematics.** • The purpose of the assignment is largely achieved. • There is a largely accurate application of relevant subject matter knowledge. • The supporting evidence is adequate; there are some acceptable, relevant examples. • The response reflects an adequate understanding of the assignment.
2	**The "2" response reflects a limited command of the relevant knowledge and skills as defined in the subject matter requirements for CSET: Mathematics.** • The purpose of the assignment is partially achieved. • There is limited accurate application of relevant subject matter knowledge. • The supporting evidence is limited; there are few relevant examples. • The response reflects a limited understanding of the assignment.
1	**The "1" response reflects little or no command of the relevant knowledge and skills as defined in the subject matter requirements for CSET: Mathematics.** • The purpose of the assignment is not achieved. • There is little or no accurate application of relevant subject matter knowledge. • The supporting evidence is weak; there are no or few relevant examples. • The response reflects little or no understanding of the assignment.
U	**The "U" (Unscorable) is assigned to a response that is unrelated to the assignment, illegible, primarily in a language other than English, or does not contain a sufficient amount of original work to score.**
B	**The "B" (Blank) is assigned to a response that is blank.**